

Quantitative Security of Block Ciphers: Designs and Cryptanalysis Tools

Thomas Baignères



PhD Defense
November 14, 2008

Prologue

Cryptography: the Basics

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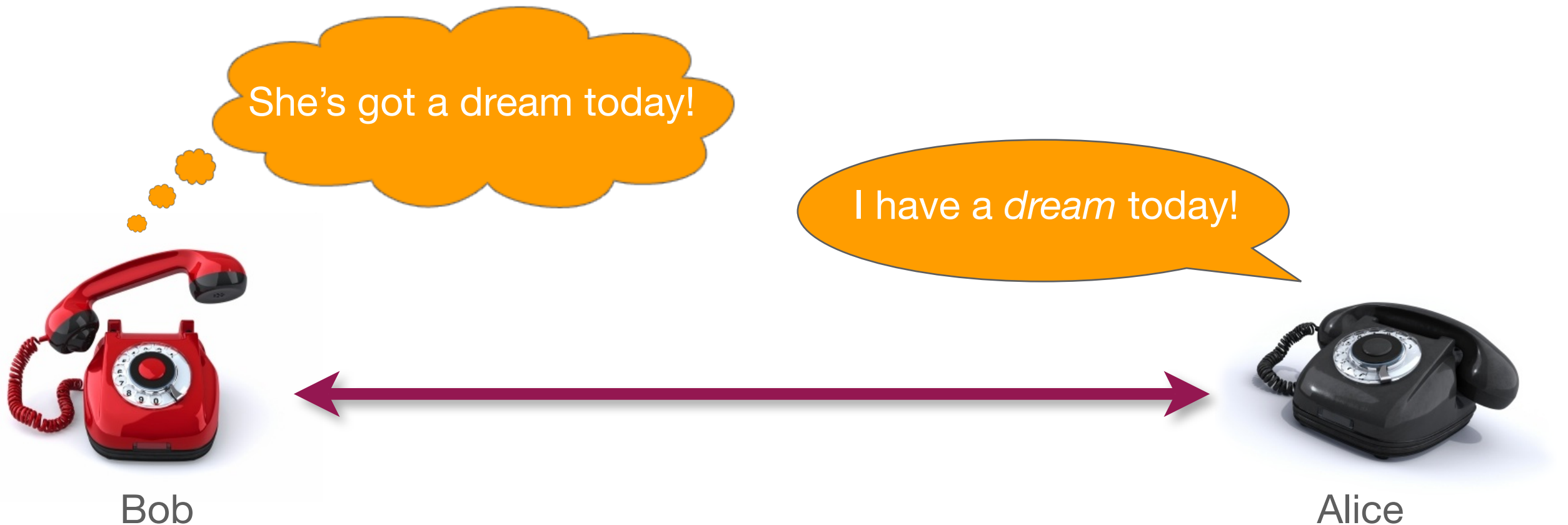
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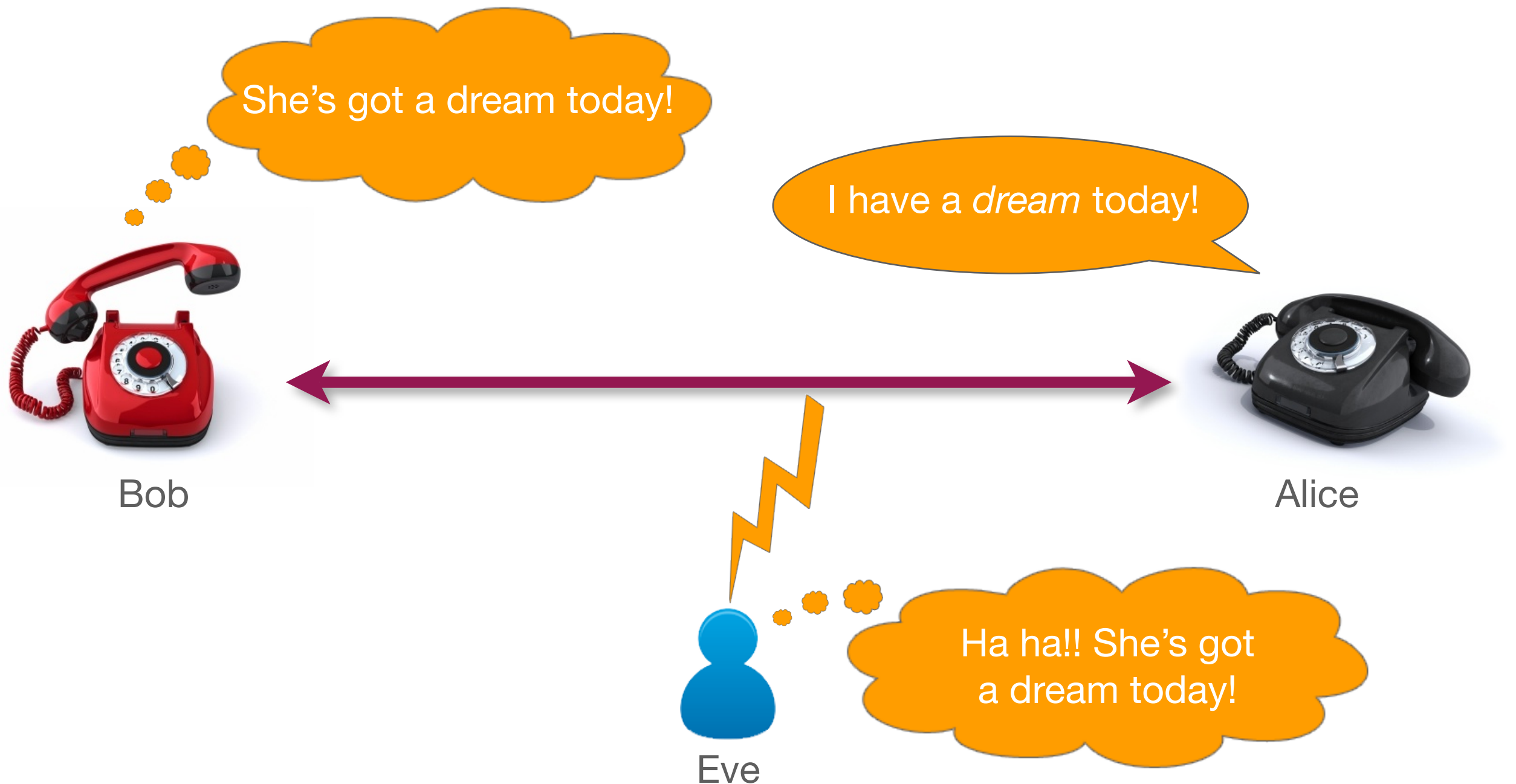
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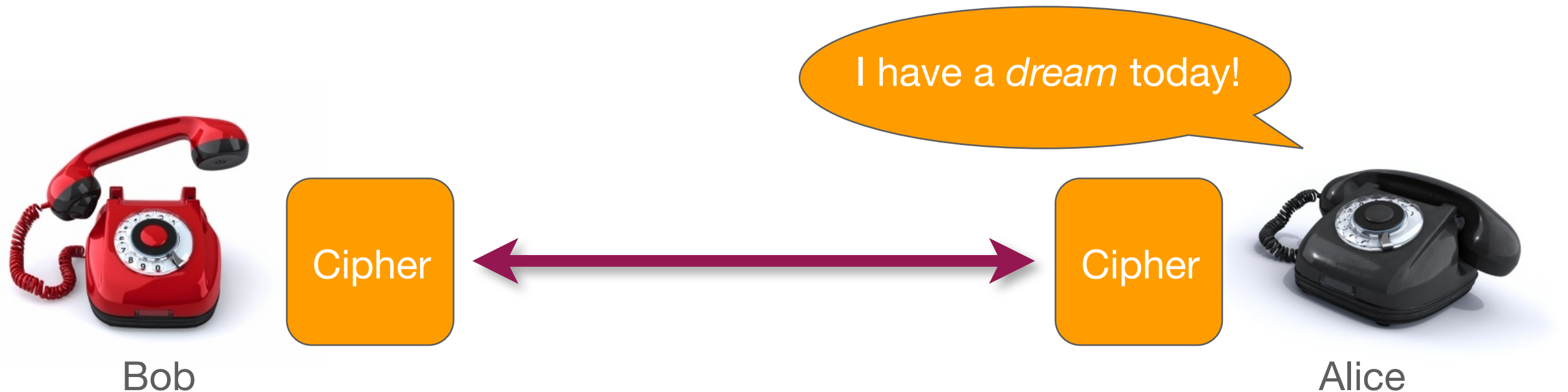
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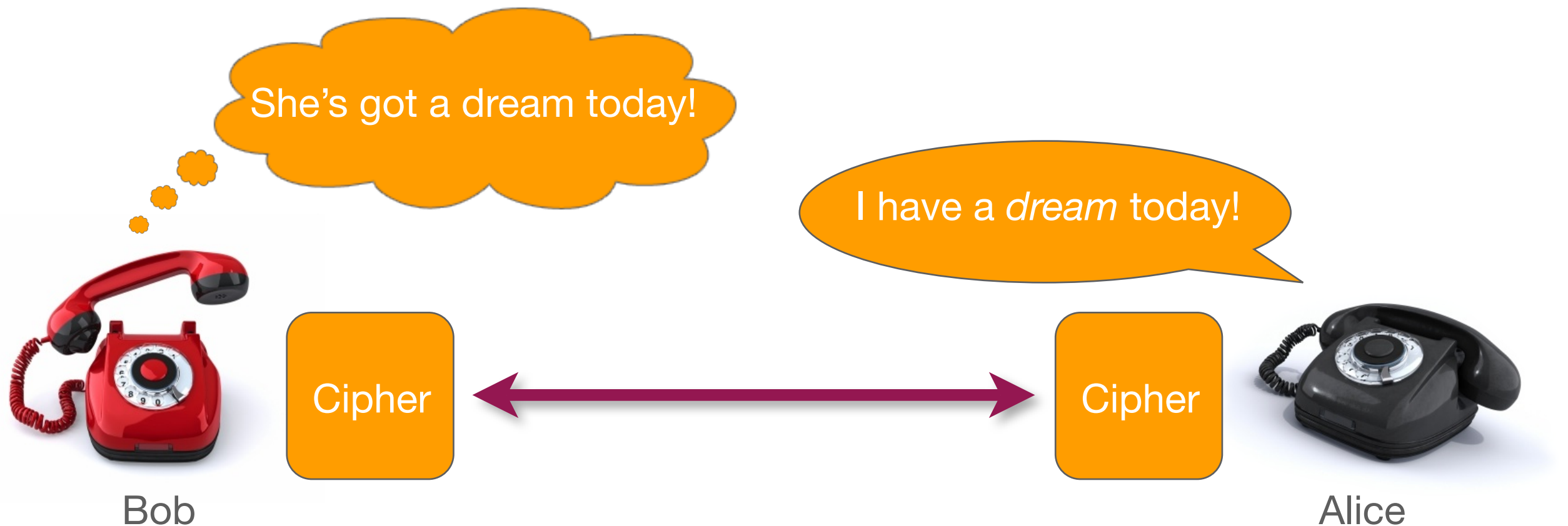
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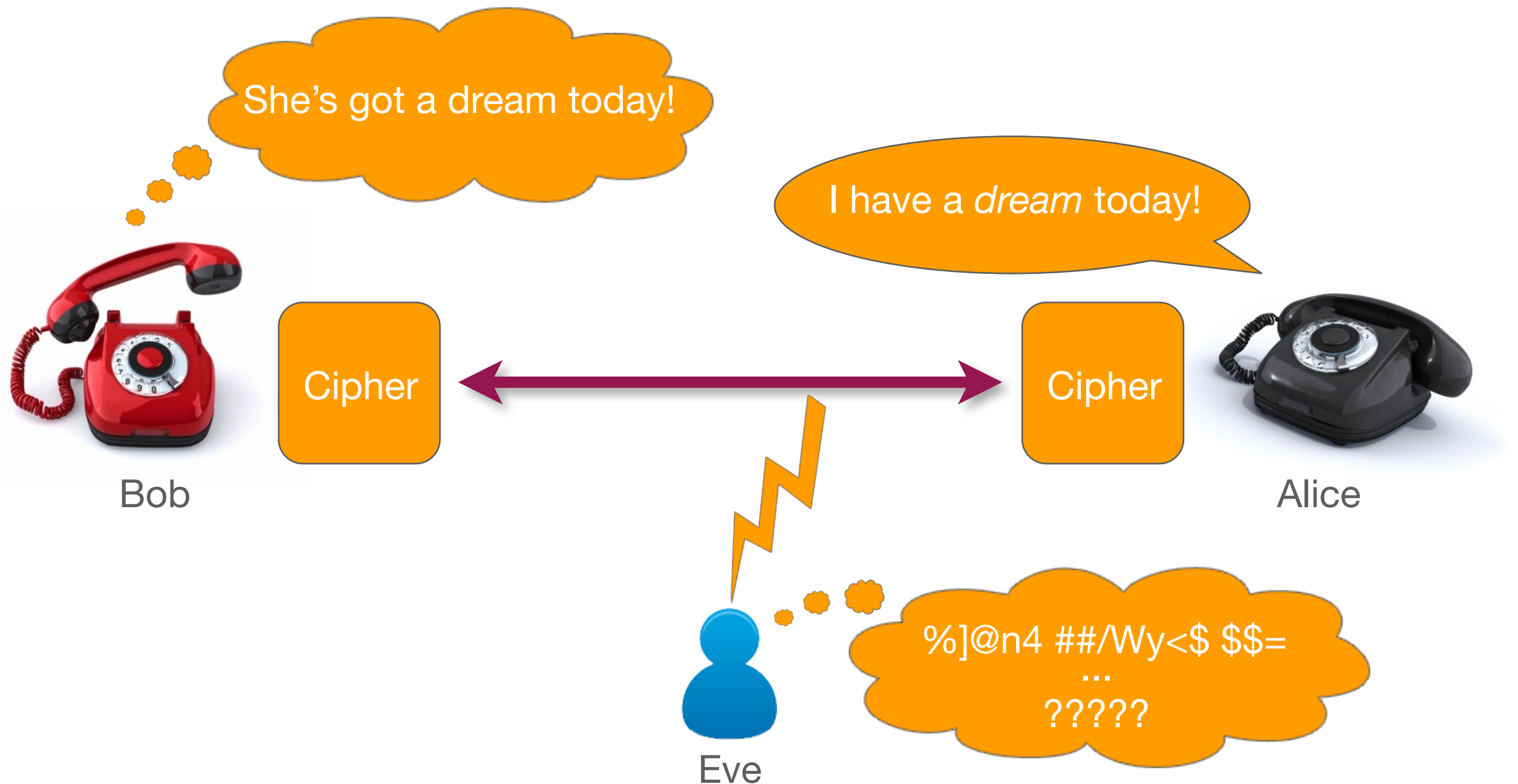
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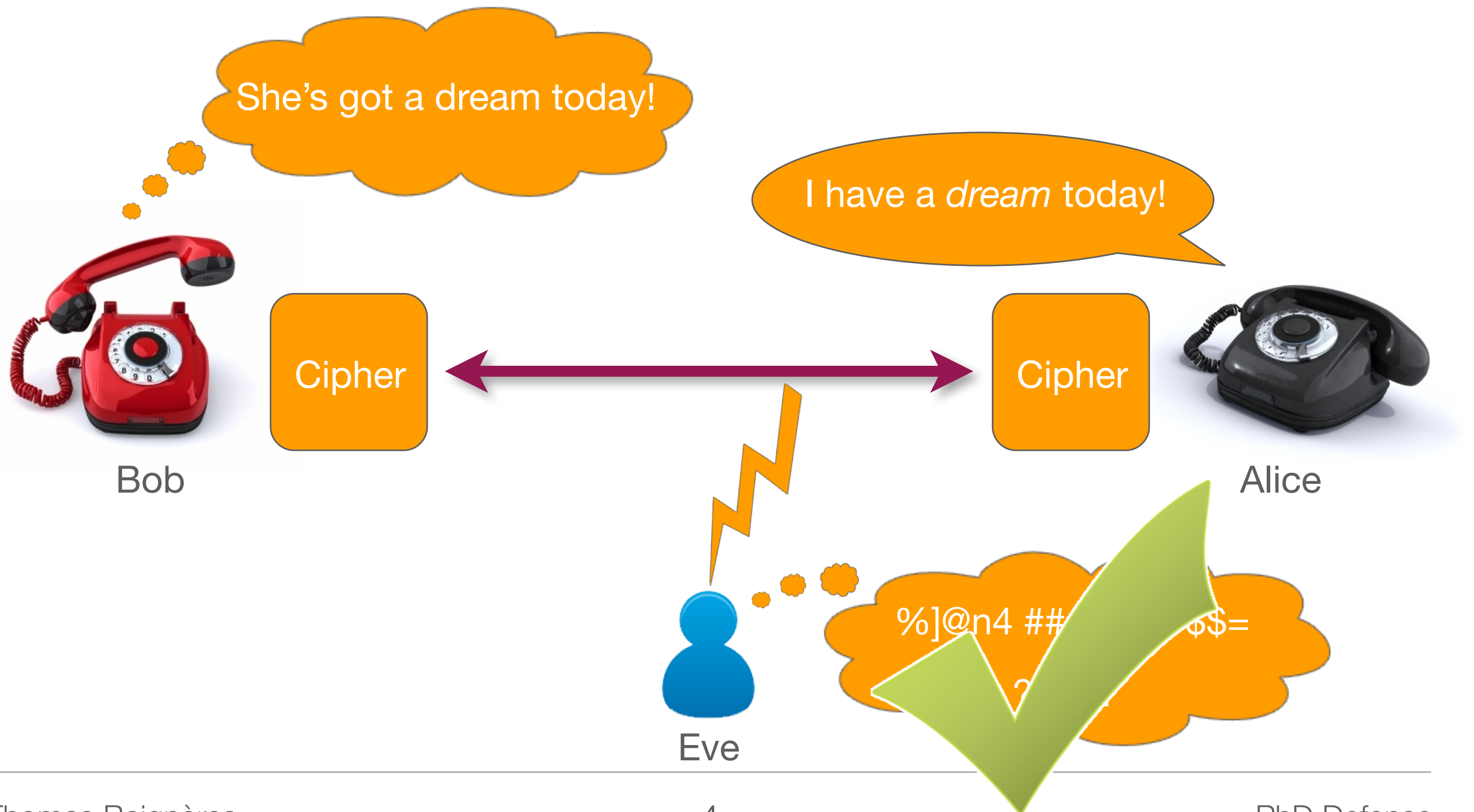
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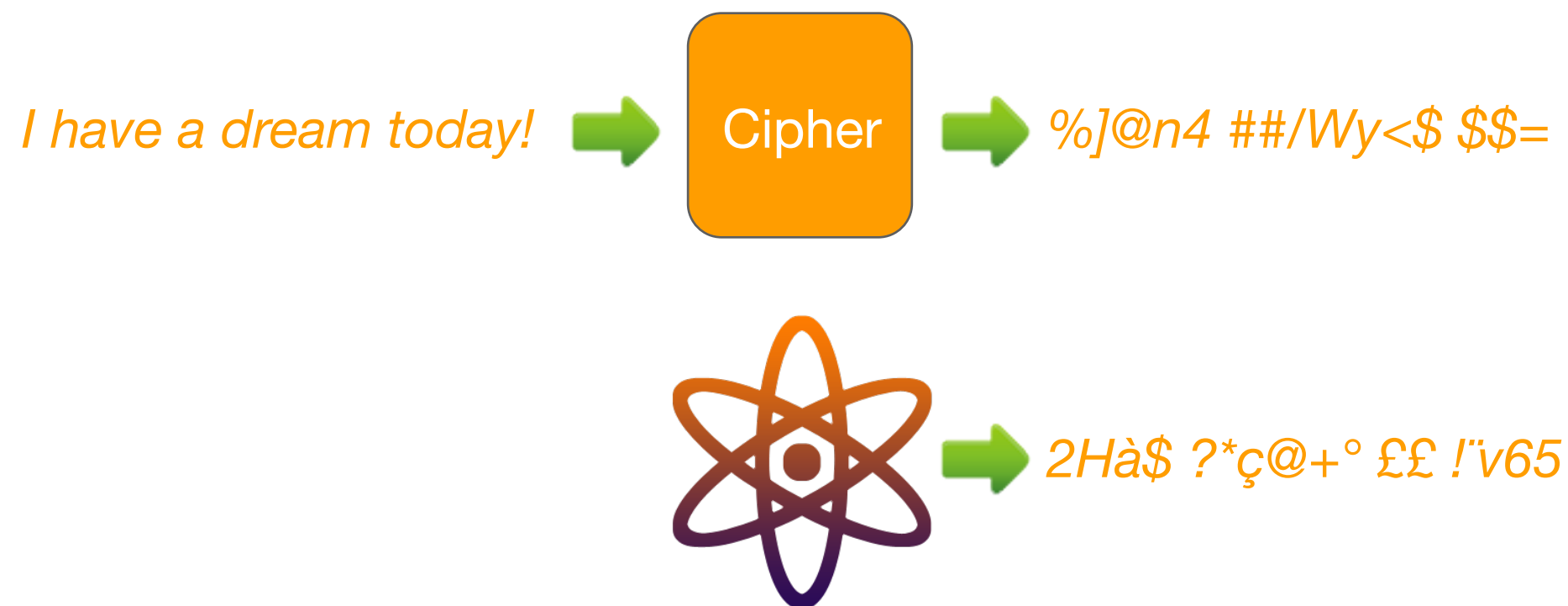
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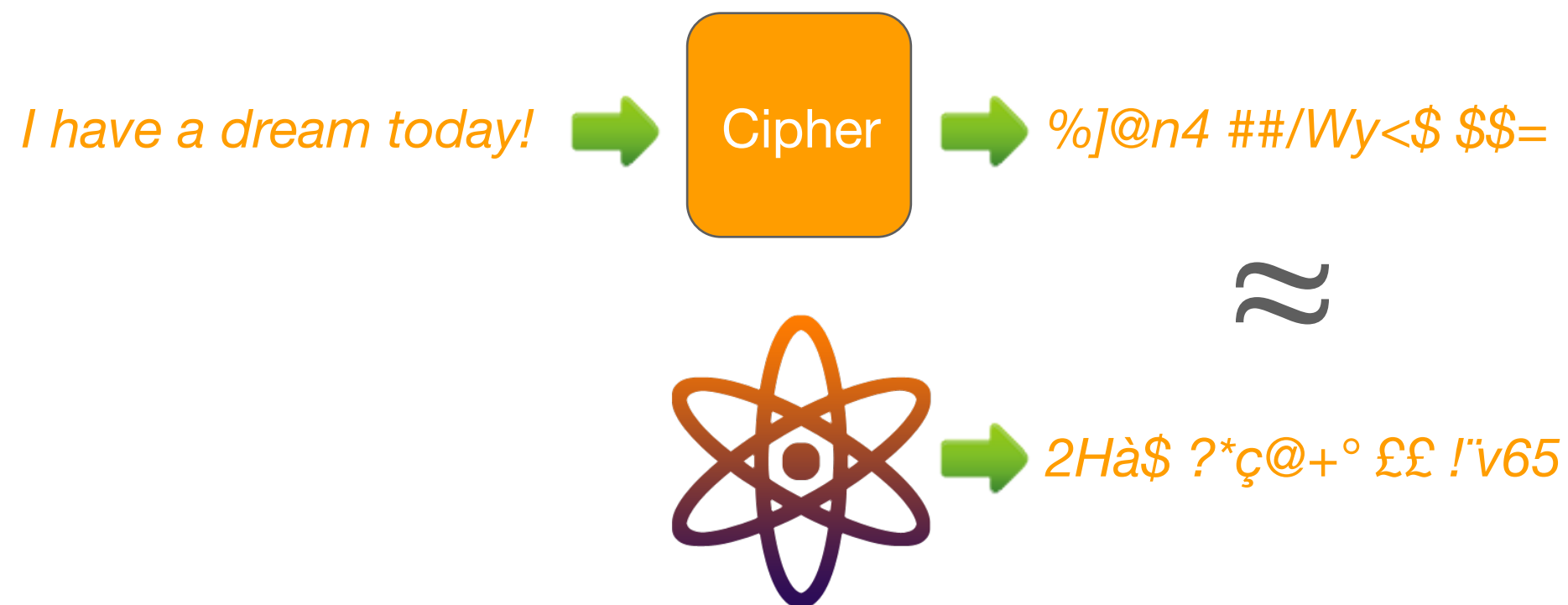
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It should be hard for Eve to guess whether she's looking at an encrypted message (ciphertext) or to pure rubbish (random string).

The security requirements in terms of a game...

... or “Cryptographers will never grow up”.

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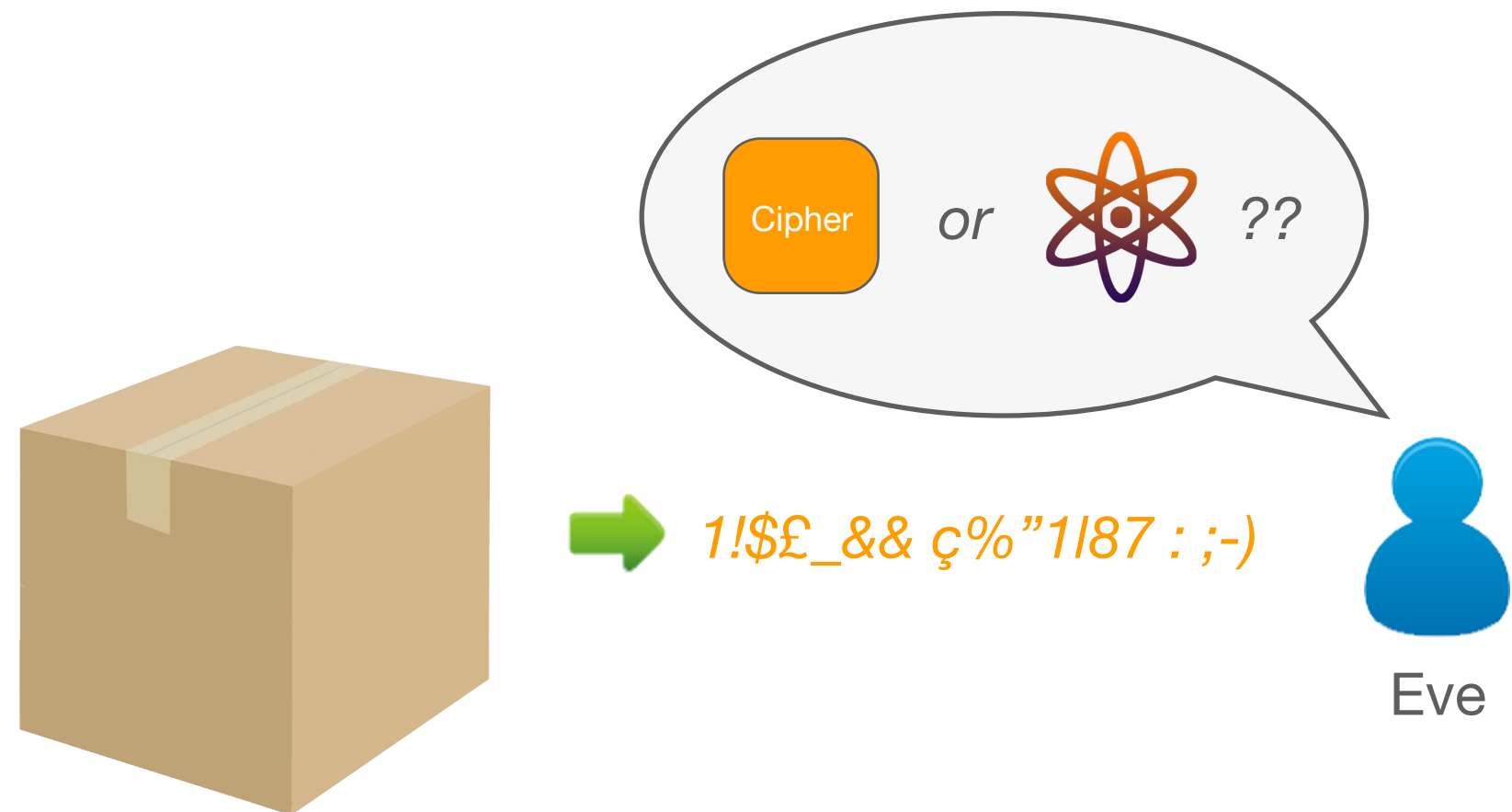
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1!\$£_&& ¸%"1/87 : ;-)

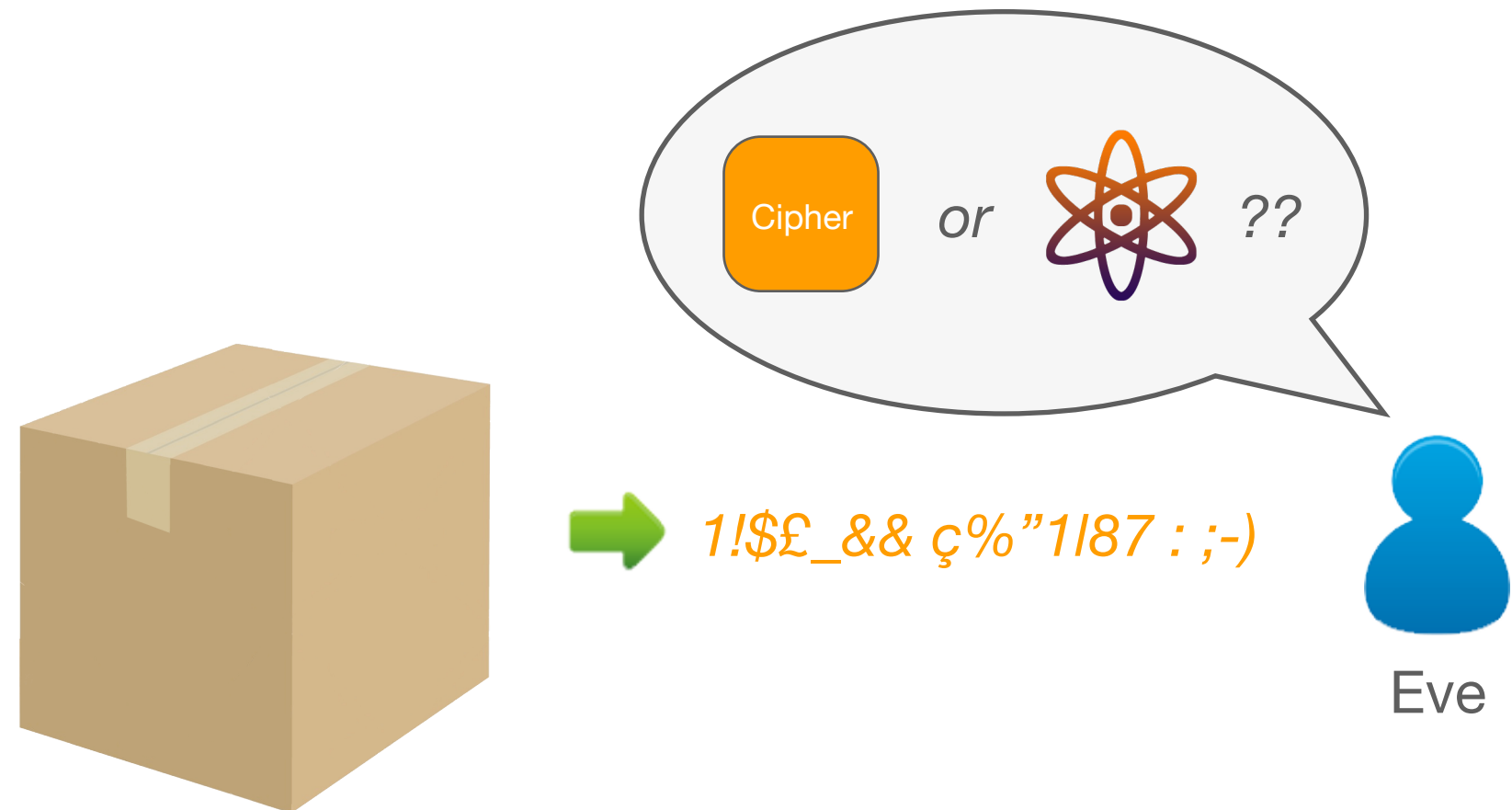
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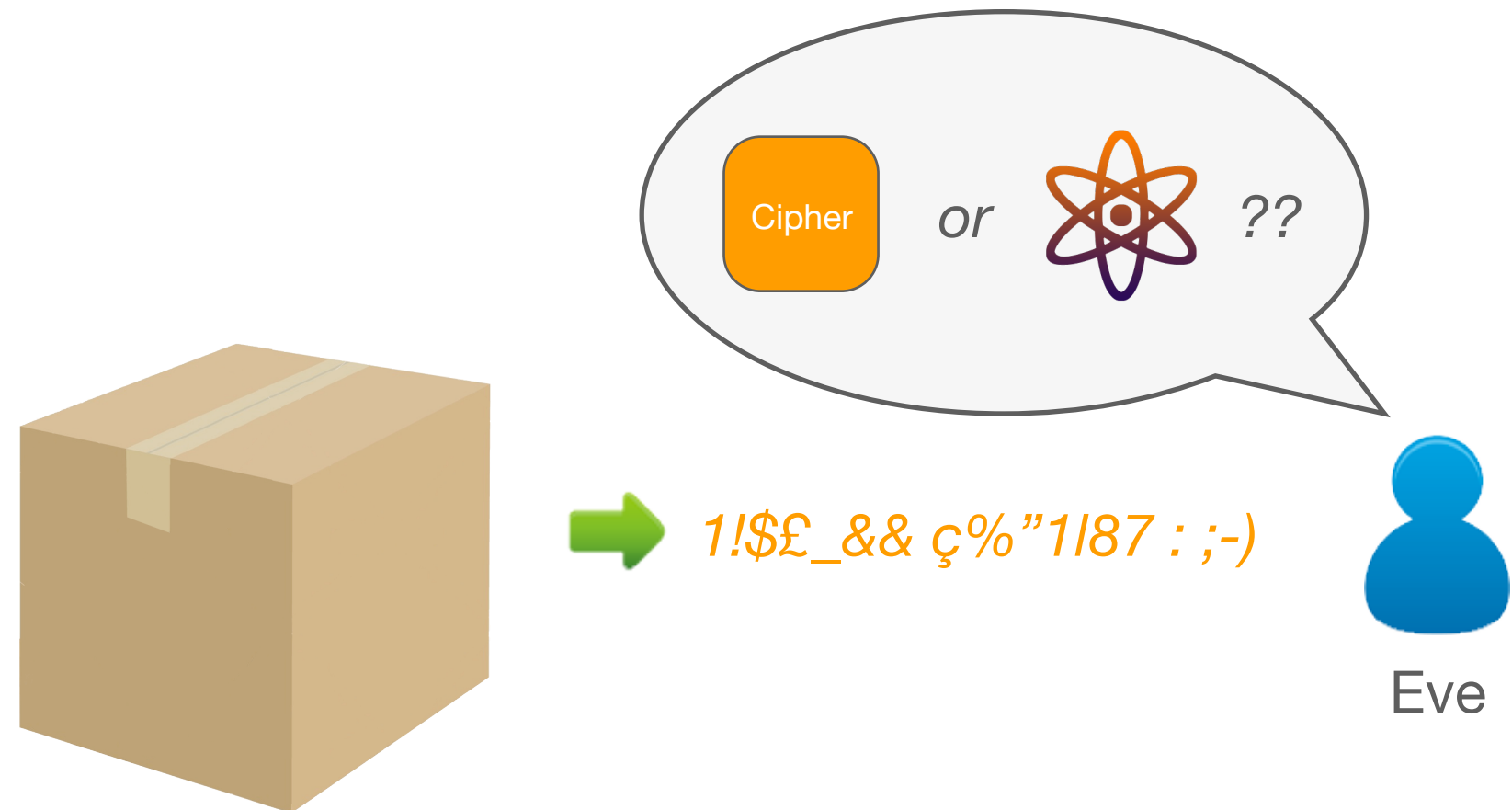
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- Eve wins if she guesses correctly.

The security requirements in terms of a game...

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- Eve wins if she guesses correctly.
- **Objective for the cryptographer:** make sure that Eve cannot do better than guessing correctly 50% of the time.

Part I: On the (In)Security of Block Ciphers: Tools for the Security Analysis

Outline

Distinguishers between two sources

Projection-based distinguishers
between two sources

Practical Implications for block ciphers

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Distinguishers between two sources

Projection-based distinguishers
between two sources

Practical Implications for block ciphers

- The game: distinguishing between two sources of randomness
- The optimal solution
- Complexity analysis: How many samples do we need to distinguish with a given efficiency?

Outline

Distinguishers between two sources



Projection-based distinguishers
between two sources

Practical Implications for block ciphers

- What if the optimal solution cannot be implemented?
- Distinguishing in practice using compression
- Example: Generalized linear distinguisher

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Distinguishers between two sources

- Cryptanalysis of SAFER K/SK

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Practical Implications for block ciphers

[BJVa04]

[BSVsac07]

[BVicits08]

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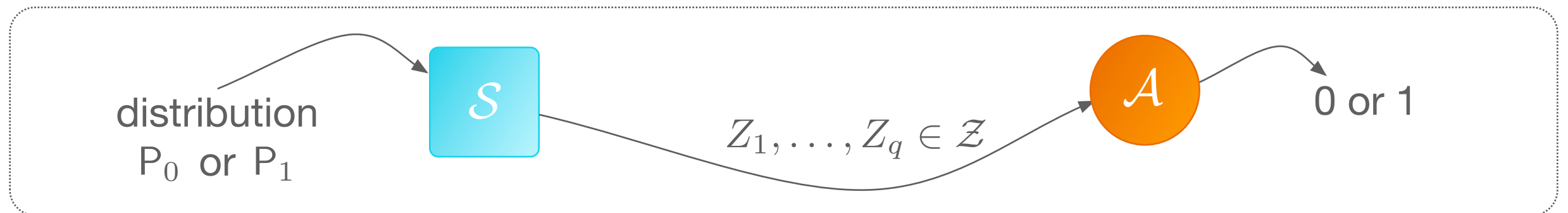
Distinguisher between two Sources

The Game

- P_0 and P_1 are two arbitrary distributions over a finite set $\mathcal{Z}$.

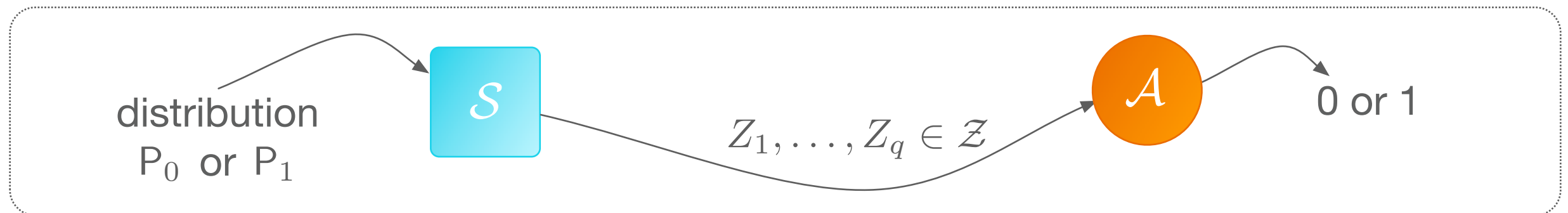
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- The ability of $\mathcal{A}$ to distinguish P_0 from P_1 is its advantage:

$$\text{Adv}_{\mathcal{A}}(P_0, P_1) = |\Pr_{P_0}[\mathcal{A}(Z_1, \dots, Z_q) = 1] - \Pr_{P_1}[\mathcal{A}(Z_1, \dots, Z_q) = 1]|$$

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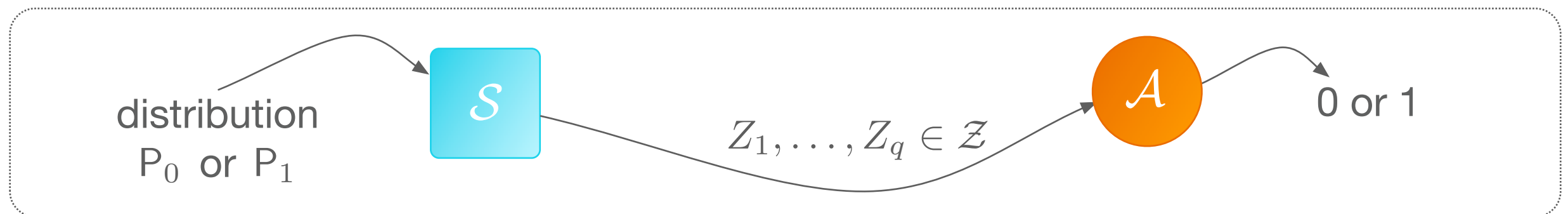
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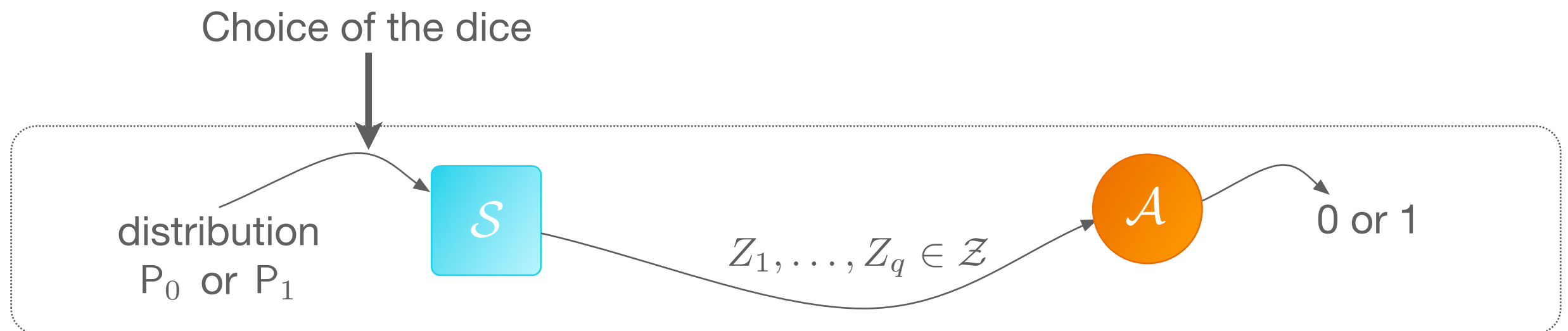


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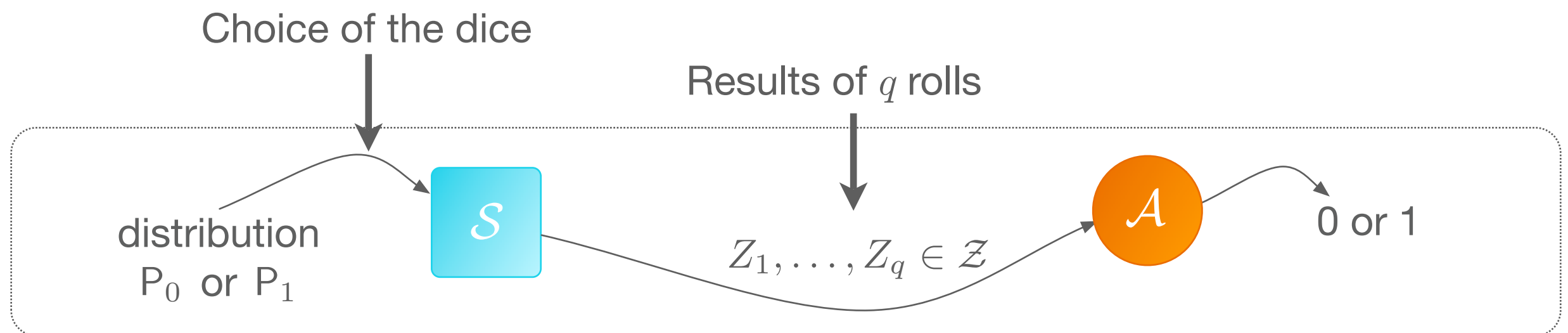


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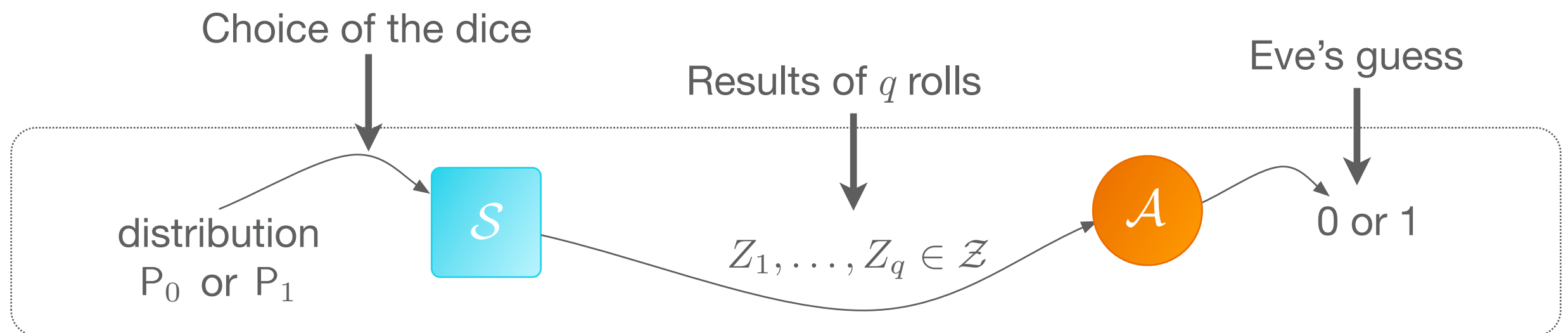


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$$D(p\|q) = \sum_{a \in \mathcal{Z}} p[a] \log \frac{p[a]}{q[a]}$$

always non-negative, 0 iff $p=q$, infinite iff $\text{Supp}(p) \not\subseteq \text{Supp}(q)$

Data Complexity Analysis

Using the **theory of types** & **Sanov's theorem** asymptotic data complexity of $\mathcal{A}^*$.

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Theorem

Let P_0 and P_1 be two distributions s.t. $\text{Supp}(P_0) \cup \text{Supp}(P_1) = \mathcal{Z}$. The advantage of $\mathcal{A}^*$ verifies

$$1 - \text{BestAdv}_q(P_0, P_1) \doteq 2^{-qC(P_0, P_1)}$$

where

$$C(P_0, P_1) = - \inf_{0 < \lambda < 1} \log \sum_{a \in \text{Supp}(P_0) \cap \text{Supp}(P_1)} P_0[a]^{1-\lambda} P_1[a]^\lambda$$

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Heuristic: $q \approx 1/C(P_0, P_1)$ allows $\mathcal{A}^*$ to reach a non-negligible advantage

is the χ^2 information between P_0 and P_1 .

Example: Biased Dice

$$P_0 = \left(\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right) \quad P_1 = \left(\frac{1}{6}, \frac{1}{6}, \frac{2}{6}, 0, \frac{1}{6}, \frac{1}{6}\right)$$

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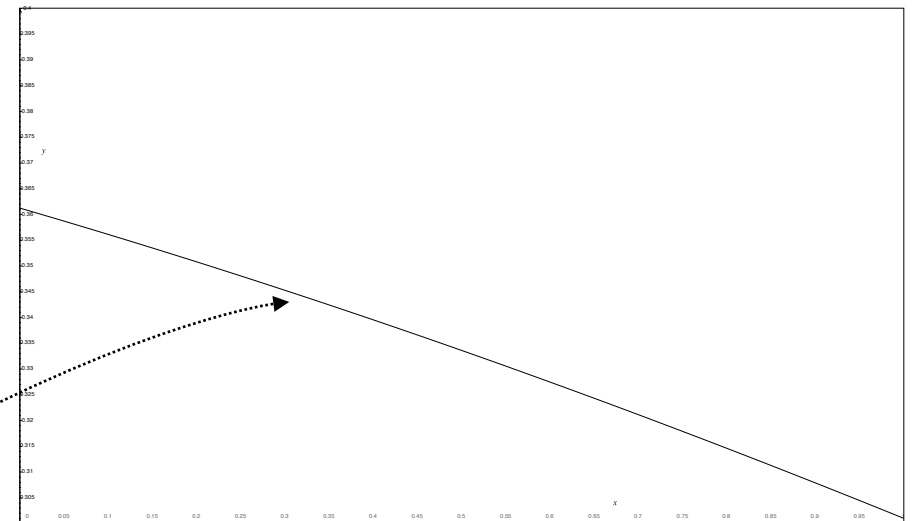
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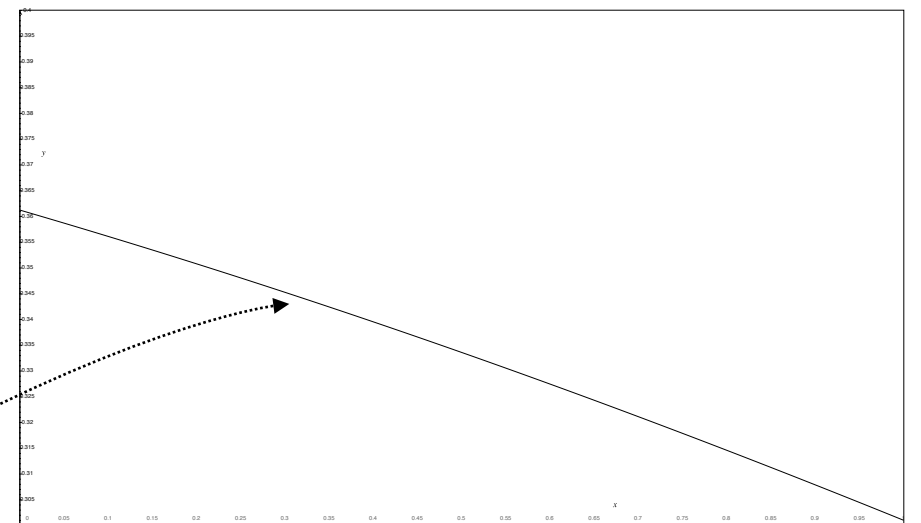


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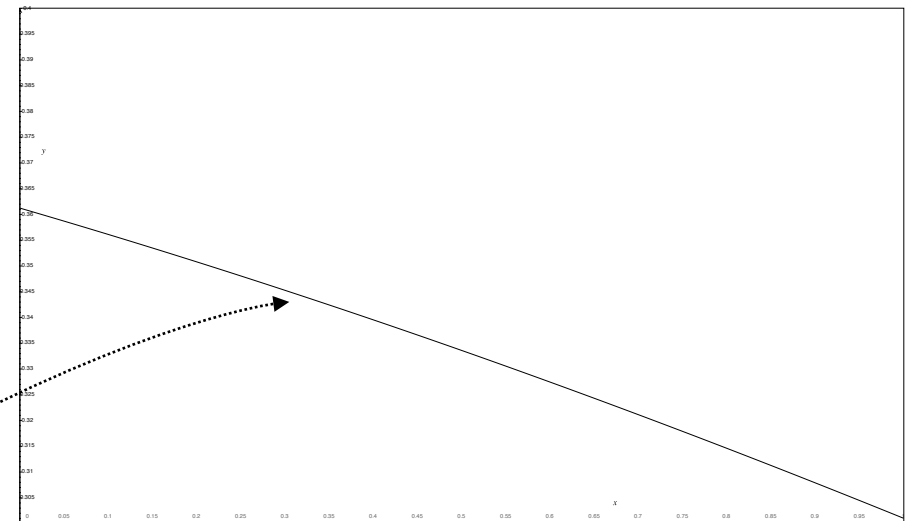


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+ approx. $\frac{1}{0.263} \approx 3.8$ queries (rolls) are sufficient to distinguish one dice from the other.

+ This is the proof that all this theory has a practical application...

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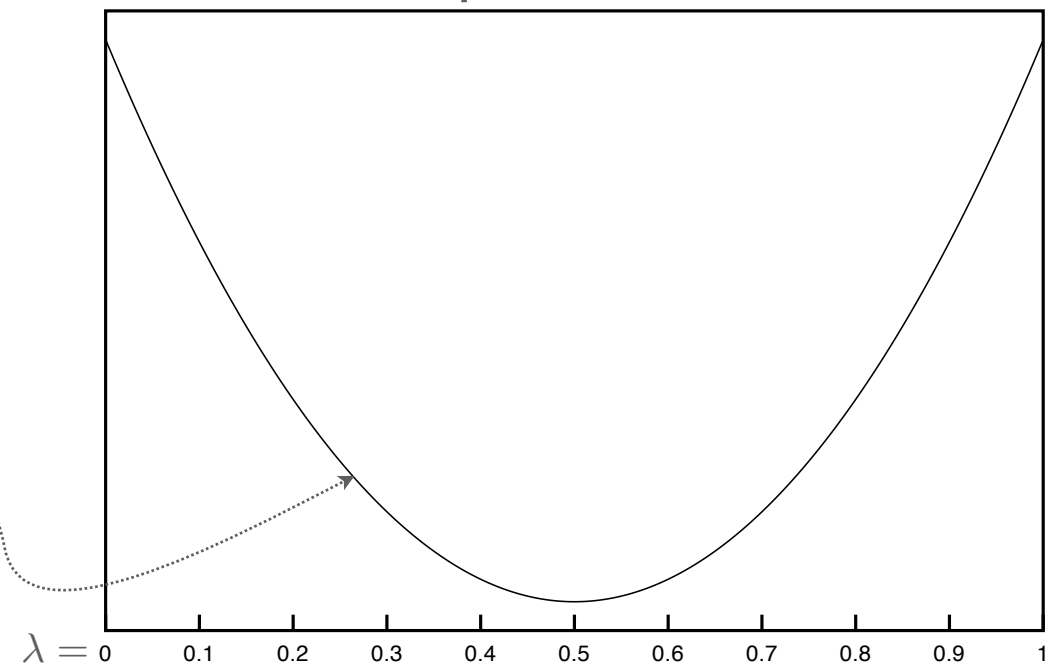


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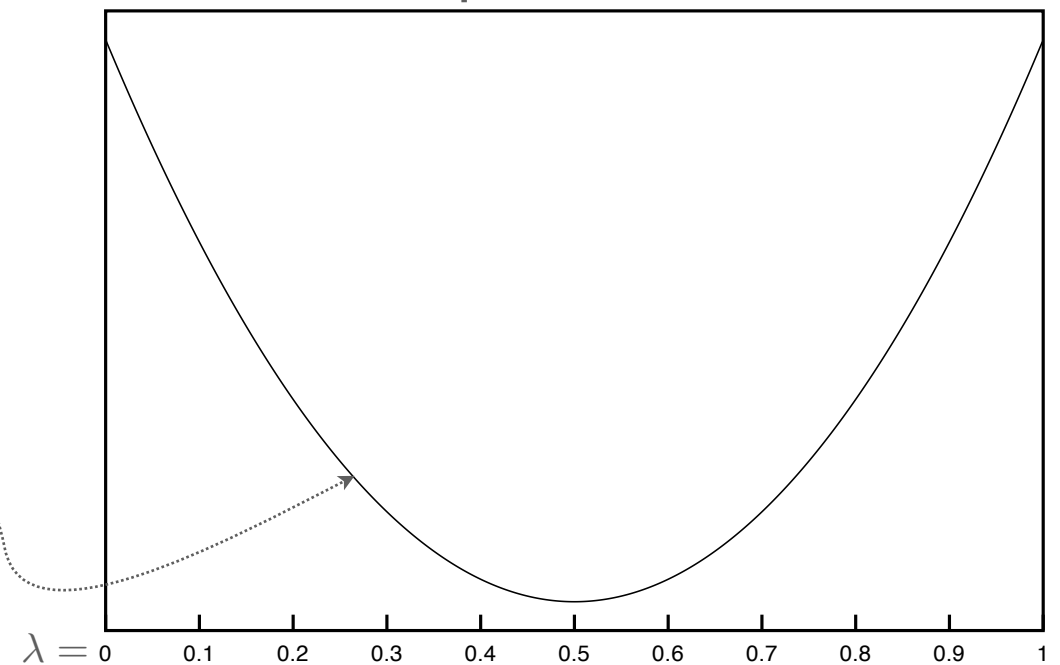
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Minimum reached for $\lambda \approx \frac{1}{2}$

$$C(P_0, P_1) \approx -\log \left(1 - \frac{\epsilon^2}{8} \right) \approx \frac{\epsilon^2}{8 \ln 2}$$

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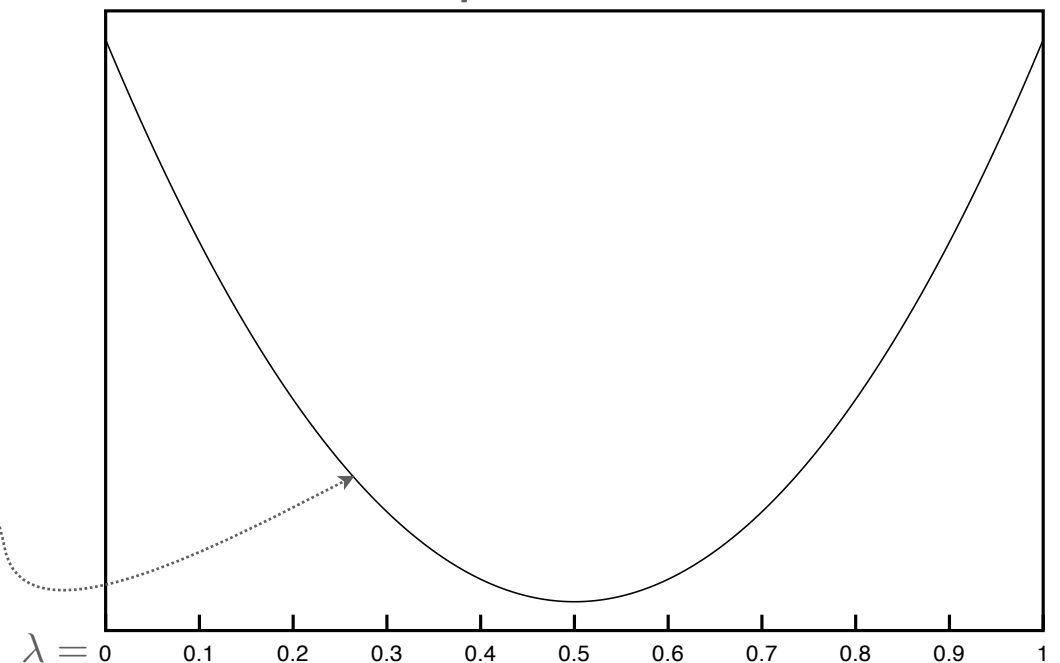
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$q \approx \frac{8 \ln 2}{\epsilon^2}$ allow to reach a non-negligible advantage.

Example with $\epsilon = 0.01$



Possible Extensions

- Case where the distributions are “close” to each other
- Case where one of the hypotheses is composite
- Case where one of the two distributions is unknown
- etc.

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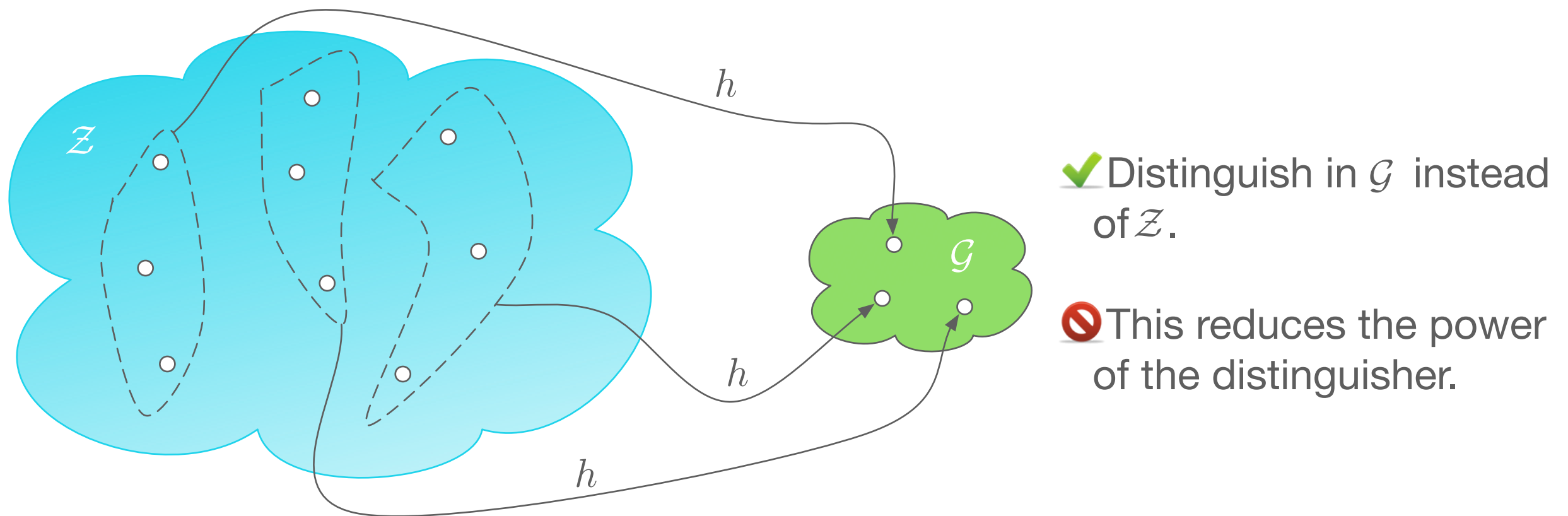
Projection Based Distinguishers

On the Need for Projection-Based Distinguishers

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- If $|\mathcal{Z}|$ is too large, the best distinguisher cannot be implemented.
- Possible solution: reduce the sample size using a **projection**:



Example: Linear Distinguishers

- $\mathcal{Z} = \{0, 1\}^n$ $\mathcal{G} = \{0, 1\}$ $P_0 = U$ $P_1 = P$ $h(Z) = a \cdot Z = a_1 Z_1 \oplus \dots \oplus a_n Z_n$
- This is a **linear distinguisher** based on the **mask** a .

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- Definition: linear probability of P:

$$\text{LP}_a(P) = \left(\mathbb{E}_P \left((-1)^{a \cdot Z} \right) \right)^2$$

Example: Linear Distinguishers



- $\mathcal{Z} = \{0, 1\}^n$ $\mathcal{G} = \{0, 1\}$ $P_0 = U$ $P_1 = P$ $h(Z) = a \cdot Z = a_1 Z_1 \oplus \dots \oplus a_n Z_n$
- This is a **linear distinguisher** based on the **mask** a .
- By implementing the optimal strategy (after the linear compression), the advantage of this linear distinguisher verifies:

$$1 - \text{Adv}(U, P) \doteq 2^{-qC(\bar{U}, \bar{P})}$$

$$a \cdot Z \sim \bar{P} \Leftrightarrow Z \sim P$$

$$a \cdot Z \sim \bar{U} \Leftrightarrow Z \sim U$$

- Definition: linear probability of P :

$$\text{LP}_a(P) = \left(\mathbb{E}_P \left((-1)^{a \cdot Z} \right) \right)^2$$

- Roughly: $C(U, P) \approx \frac{\text{LP}_a(P)}{8 \ln 2}$  $q \approx \frac{8 \ln 2}{\text{LP}_a(P)}$ are enough (well known...)

Extending the Notion of Linear Probability

- The previous example only works for sets of the form $\mathcal{Z} = \{0, 1\}^n$.
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- Example: when $\mathcal{Z} = \{0, 1\}^n$ we have $\chi(a) = (-1)^{u \cdot a}$ for some u
- Consequence: when $\mathcal{Z} = \{0, 1\}^n$ this new definition corresponds to the old one!

Lin. Dist. for Sources over Arbitrary Sets

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Lemma 7.5 *Let P_0 be the uniform distribution on a finite subgroup H of $\mathbf{C}^\times$ of order d . Let $\mathcal{D} = \{P_u : u \in H\}$ be a set of d distributions on H defined by (7.10). The q -limited distinguisher between the null hypothesis $H_0 : P = P_0$ and the alternate hypothesis $H_1 : P \in \mathcal{D}$ defined by the distribution acceptance region $\Pi_q^* = \Pi^* \cap \mathcal{P}_q$, where*

$$\Pi^* = \left\{ P \in \mathcal{P} : \|P\|_\infty \geq \frac{\log(1 - \epsilon)}{\log(1 - \epsilon) - \log(1 + (d - 1)\epsilon)} \right\}, \quad (7.11)$$

is asymptotically optimal and its advantage BestAdv_q is such that

$$1 - \text{BestAdv}_q(H_0, H_1) \doteq 2^{q \inf_{0 < \lambda < 1} \log \frac{1}{d} ((1 + (d - 1)\epsilon)^\lambda + (d - 1)(1 - \epsilon)^\lambda)}.$$

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$$\inf_{\Pi^*} \sup_{P \in \mathcal{D}} \|\Pi^* P\|_\infty \geq \frac{\log(1 - \epsilon)}{\log(1 - \epsilon) - \log\left(\frac{1}{d} \sum_{u \in H} (1 - \epsilon)^{\lambda_u}\right)} \quad (7.11)$$

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Which shows how to use the generalized LP to build a linear distinguisher over arbitrary sets...

and allows to conclude that a linear distinguisher needs $q \approx \frac{8 \ln 2}{(d-1) \text{LP}_\infty(P)}$ to reach a good advantage.

Part I: On the (In)Security of Block Ciphers:
Tools for the Security Analysis



Practical Implications for Block Ciphers

Applications on SAFER K/SK

- We attack SAFER with a $\boxplus$ -linear cryptanalysis.
- Use the toolbox to find characteristics within SAFER K/SK.
- To compute the complexities we consider several characteristics among the hull (i.e., all characteristics share the same input/output characters).
- To turn distinguishing attacks into key recovery attacks, we also take advantage of the linearity of the key schedule.

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Nbr Rounds	Complexity
2	$2^{23} / 2^{31}$
3	2^{38}
4	2^{49}
5	2^{56}

Other Applications

- Two new Digital Encryption Algorithm for Numbers (based on the AES): DEAN18 and DEAN27 which respectively encrypts blocks made of 18 and 27 decimal digits.
- Resistance against our generalization of linear cryptanalysis.
- New attacks on TOY100 (toy cipher that encrypts blocks of 32 decimal digits).
- Break 9 (10 ?) rounds out of 12.

Part II: Designs and Security Proofs

Outline

Block Ciphers

Dial **C** for Cipher

KFC: the Krazy Feistel Cipher

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KFC: the Krazy Feistel Cipher

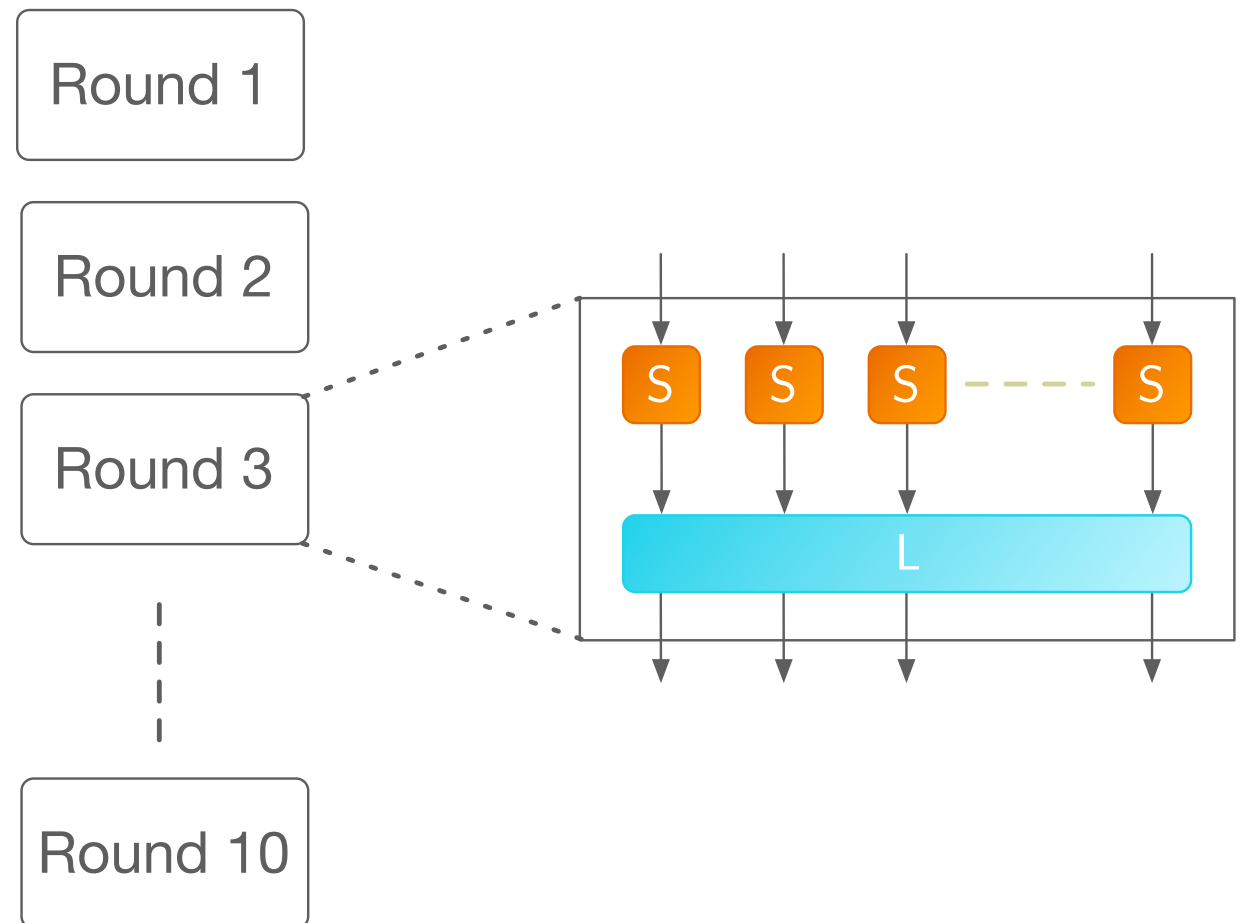
- The Luby-Rackoff Model
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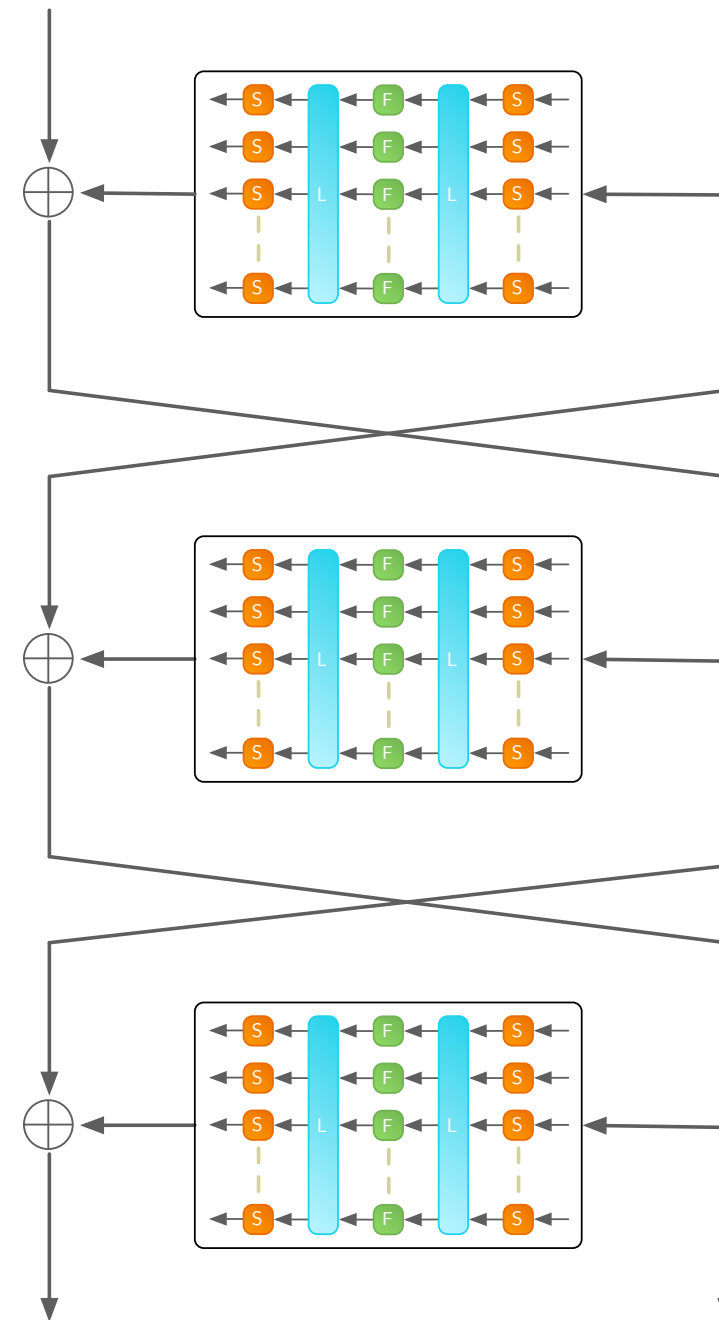


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[BV_{sac}05]

[BF_{sac}06]

[BF_a06]

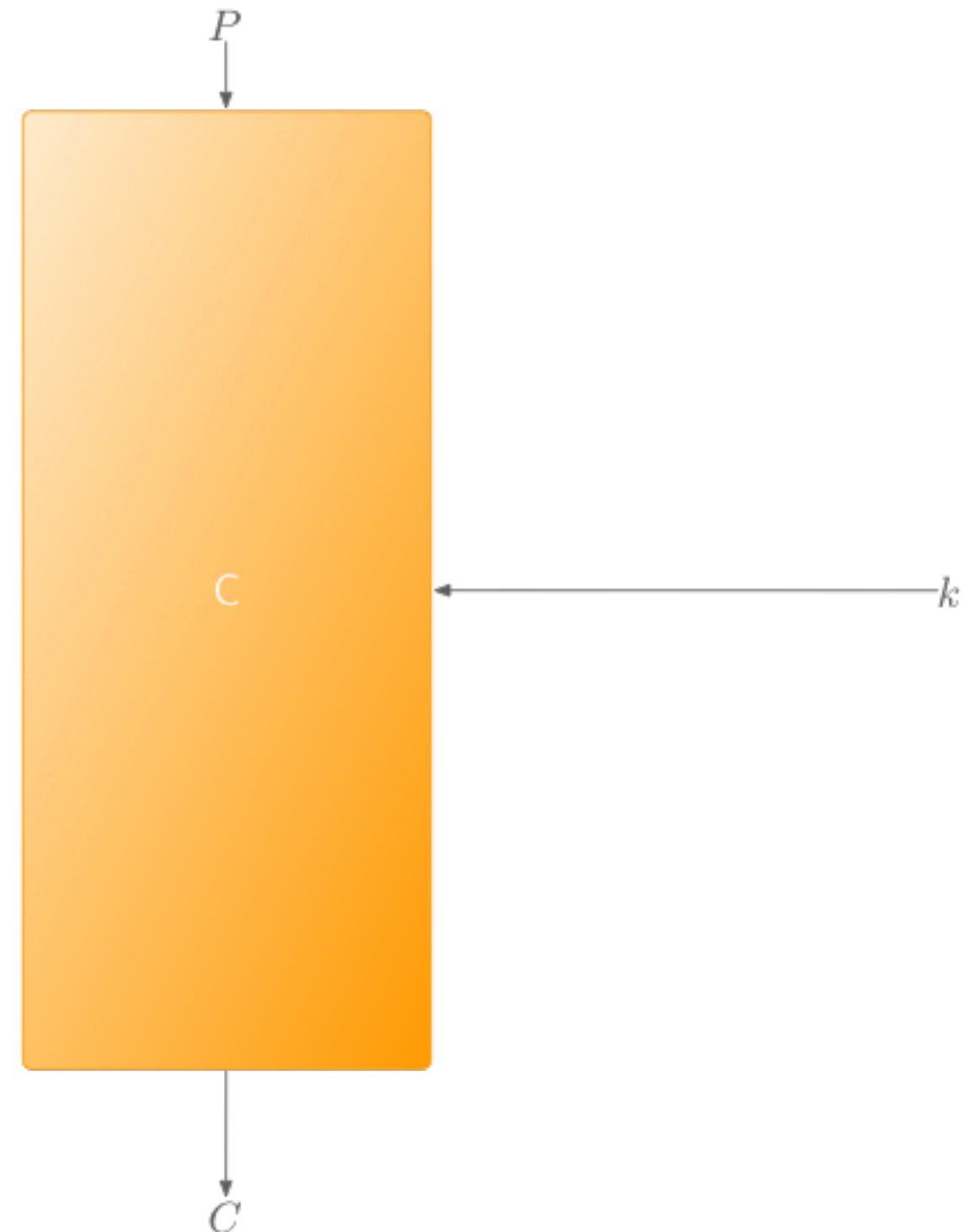
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Block Ciphers

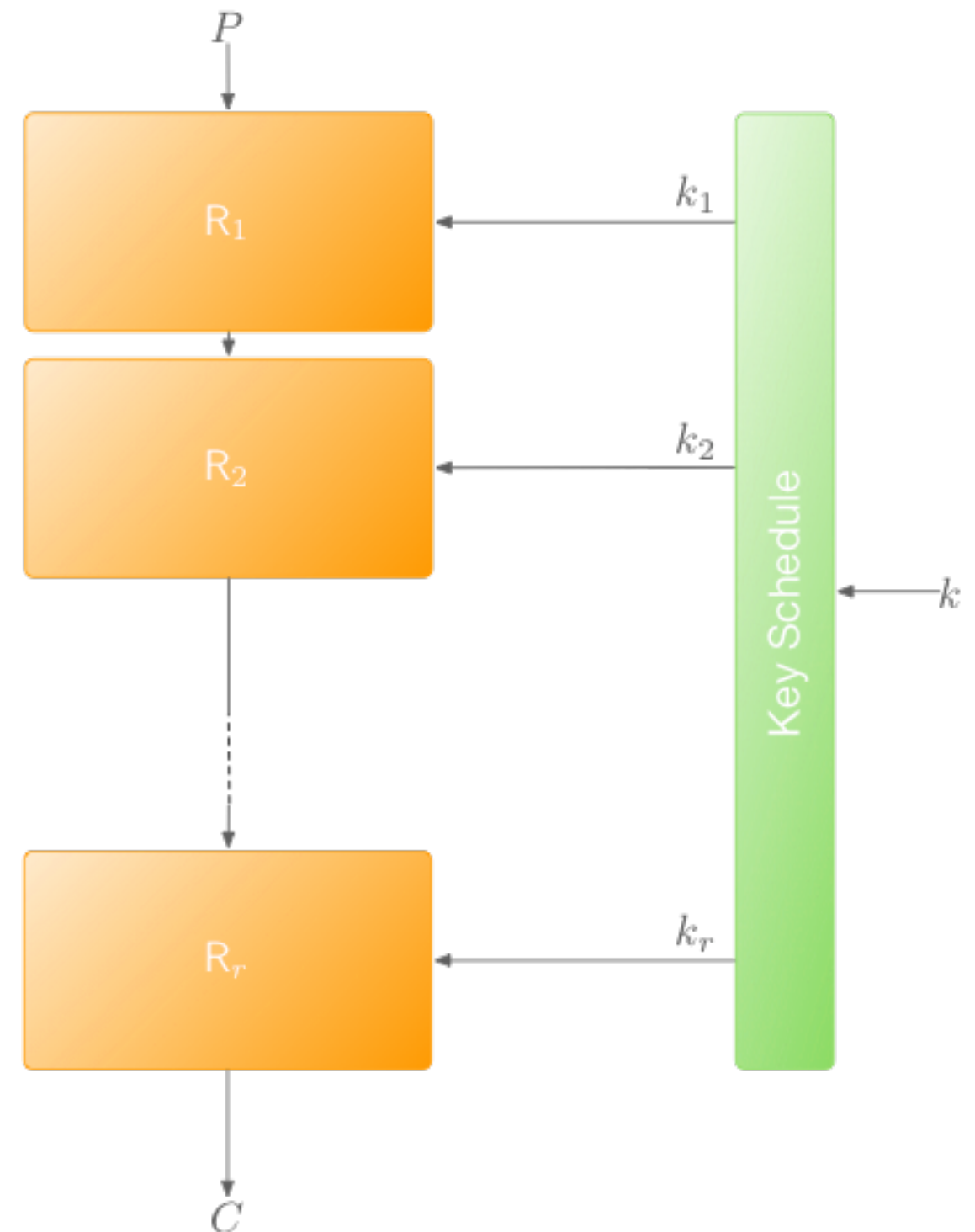
A Typical Iterated Block Cipher

- A block cipher on a finite set is a family of permutations on that set, indexed by a parameter call the key.



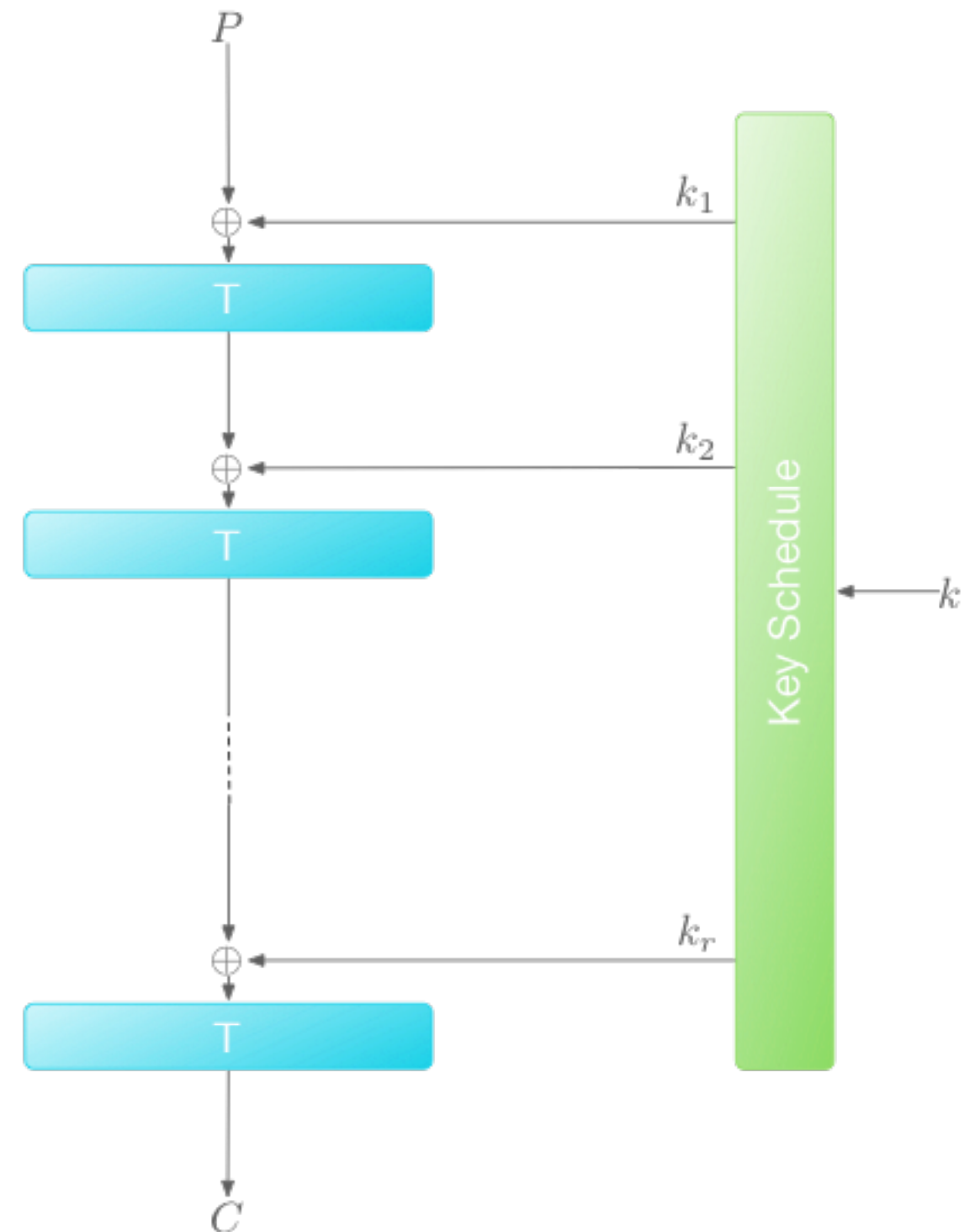
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- Each round is parameterized by a key derived from the main secret key by means of a Key Schedule.
- Usually, the rounds all share the same design, e.g., a round key addition followed by a fixed (nonlinear) transformation.



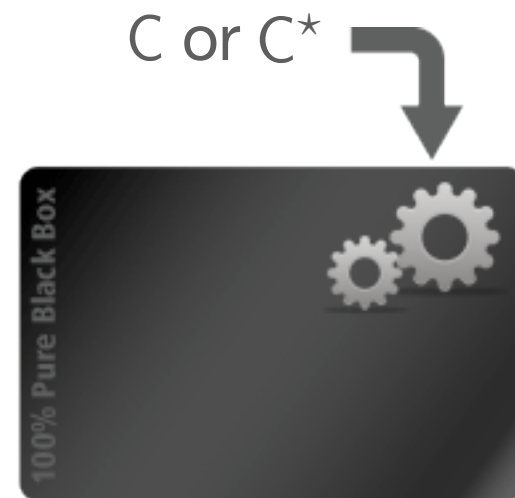
What Should we Expect from a Block Cipher?

It should be **fast** and **secure**!



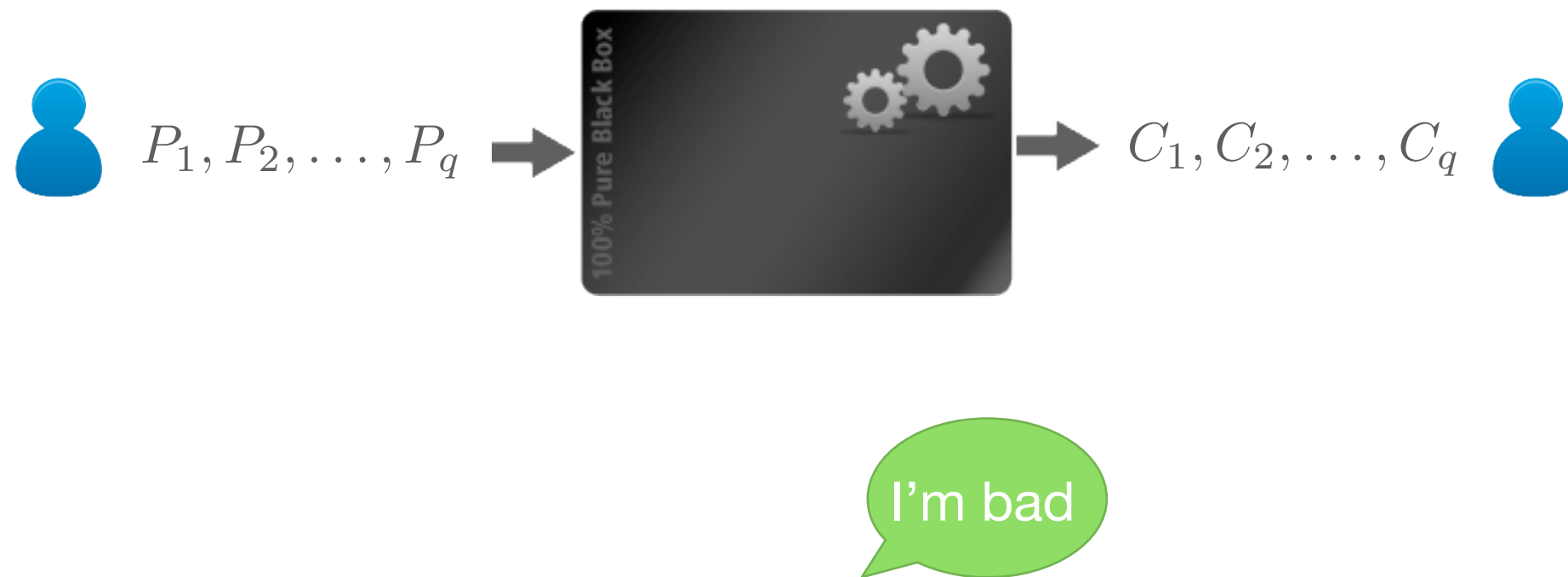
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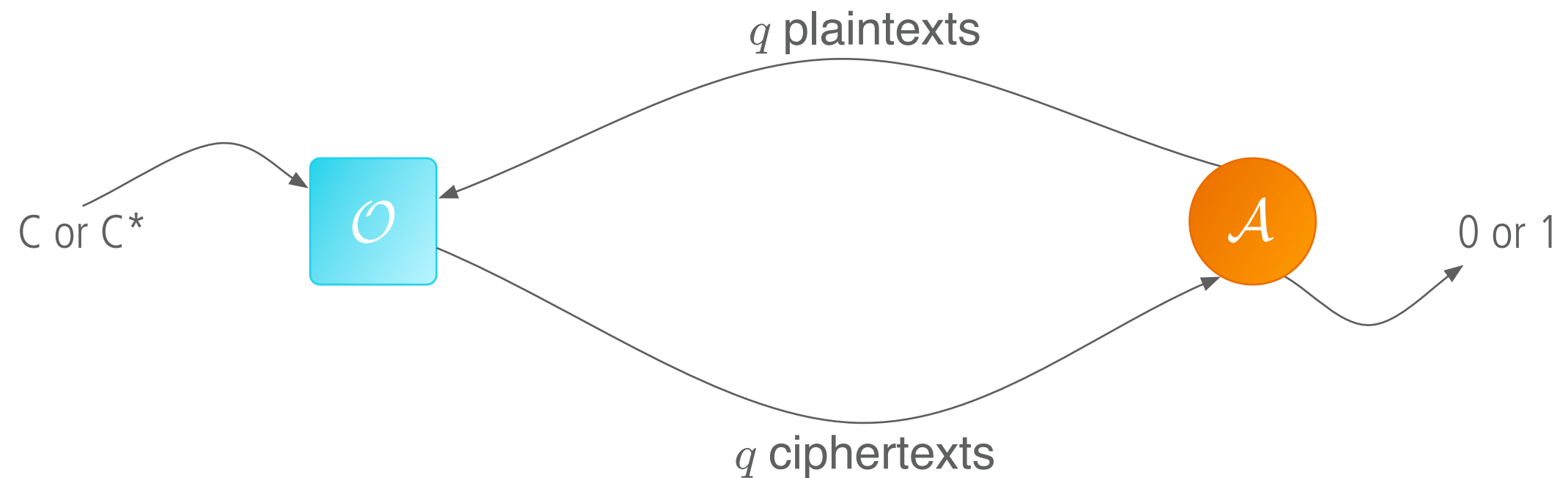
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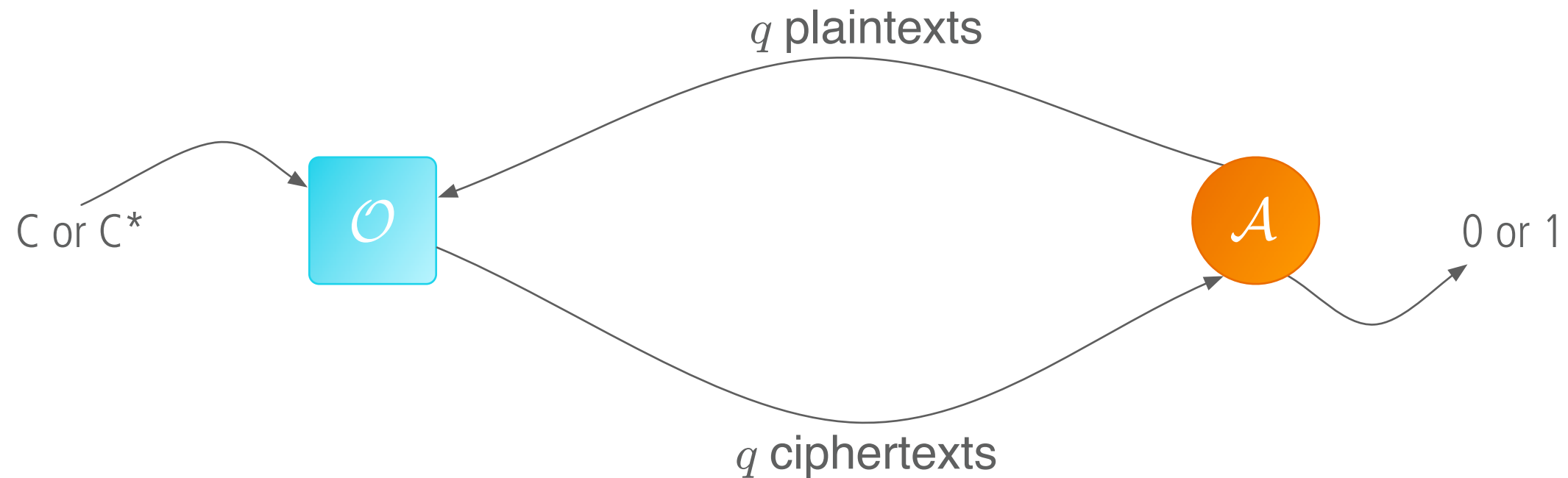
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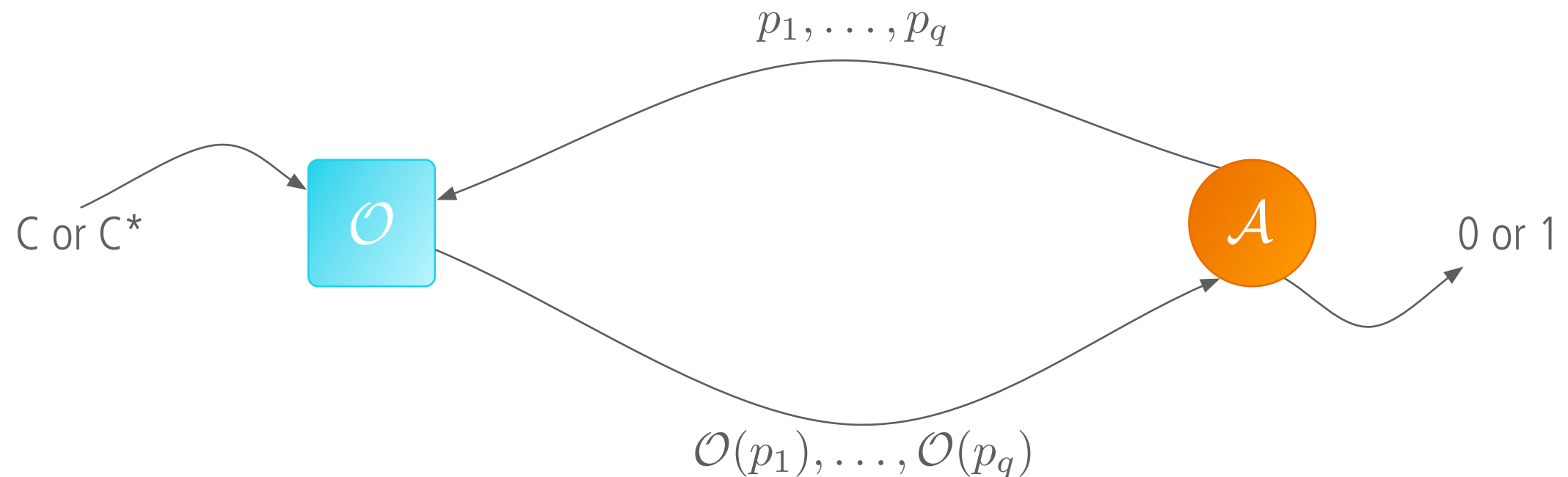
$$\text{Adv}_{\mathcal{A}}(\mathcal{C}, \mathcal{C}^*) = |\Pr[\mathcal{A}(\mathcal{C}) = 1] - \Pr[\mathcal{A}(\mathcal{C}^*) = 1]|$$

Advantage of the q -limited adversary $\mathcal{A}$ between $\mathcal{C}$ and $\mathcal{C}^*$

✓ The block cipher $\mathcal{C}$ is secure if the advantage of $\mathcal{A}$ is negligible for all $\mathcal{A}$'s.

The Luby-Rackoff Model

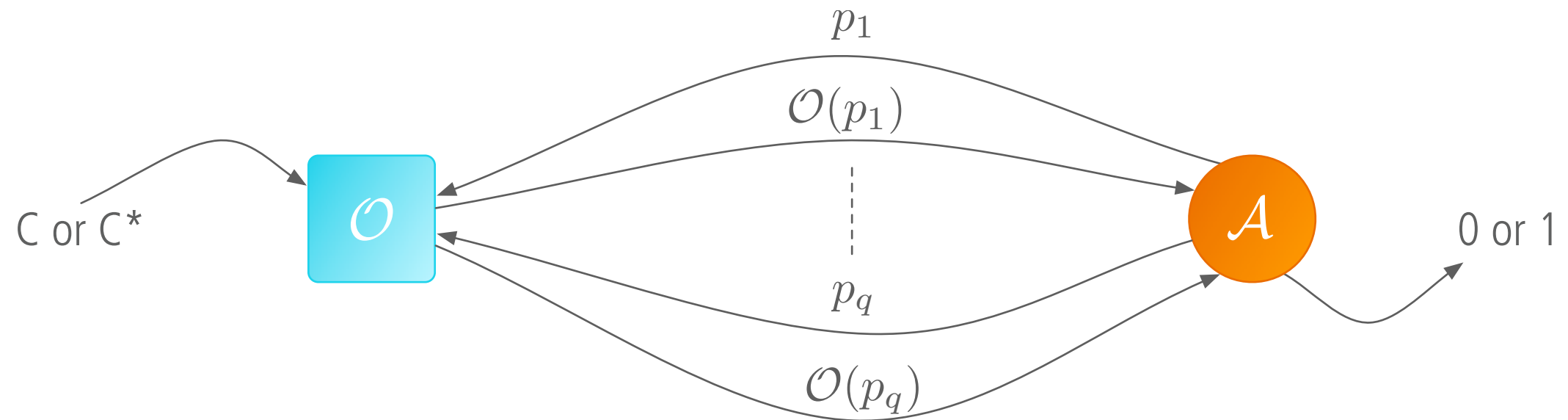
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$\mathcal{A}$ is **non-adaptive** if the q plaintexts are chosen “at once”.

The Luby-Rackoff Model

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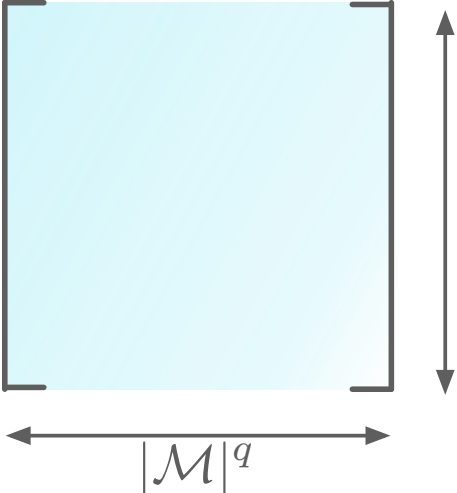


$\mathcal{A}$ is **adaptive** if plaintext i depends on ciphertexts $1, \dots, i - 1$.

Computing $\text{Adv}_{\mathcal{A}}(\mathcal{C}, \mathcal{C}^*)$

- Computing the advantage is not a trivial task in general.
- Possible solution: use Vaudenay's Decorrelation Theory.

$$\max_{\mathcal{A}} \text{Adv}_{\mathcal{A}}(\mathcal{C}, \mathcal{C}^*) = \frac{1}{2} \|\llbracket \mathcal{C} \rrbracket^q - \llbracket \mathcal{C}^* \rrbracket^q\|$$

$$\llbracket \mathcal{C} \rrbracket^q =$$


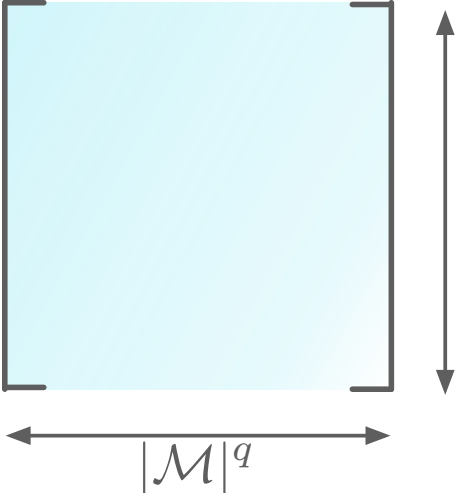
$$|\mathcal{M}|^q = 2^{128 \cdot q} \text{ for a 128-bits block cipher}$$

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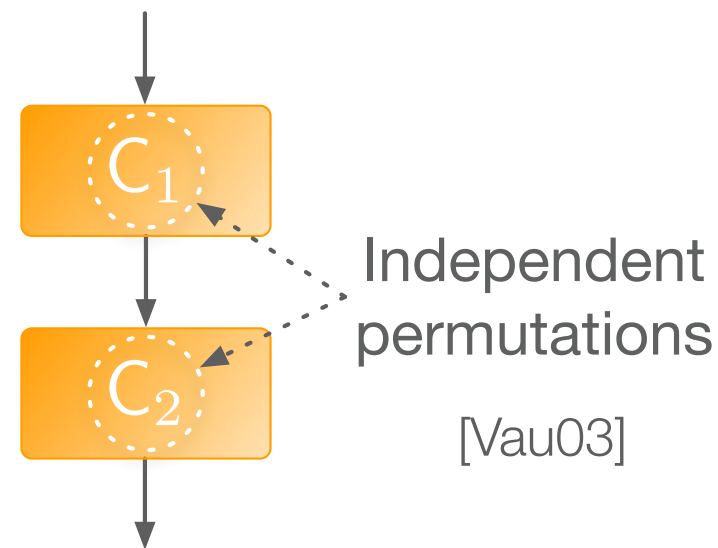
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Tricks for Computing $\text{Adv}_{\mathcal{A}}(\mathcal{C}, \mathcal{C}^*)$

To deal with the size of the distribution matrices:

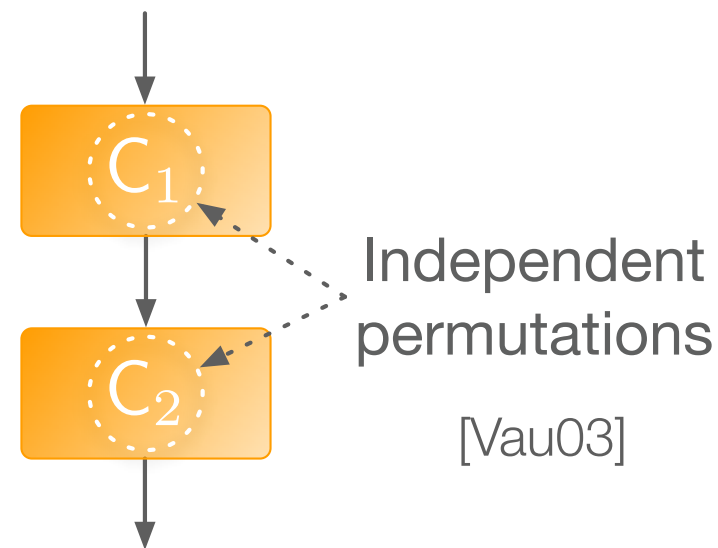
$\text{+ } [\mathcal{C}_2 \circ \mathcal{C}_1]^q = [\mathcal{C}_1]^q \times [\mathcal{C}_2]^q$



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+ Take advantage of the symmetries of the block cipher in order to compute the distribution matrix of each round

Part II: Designs and Security Proofs

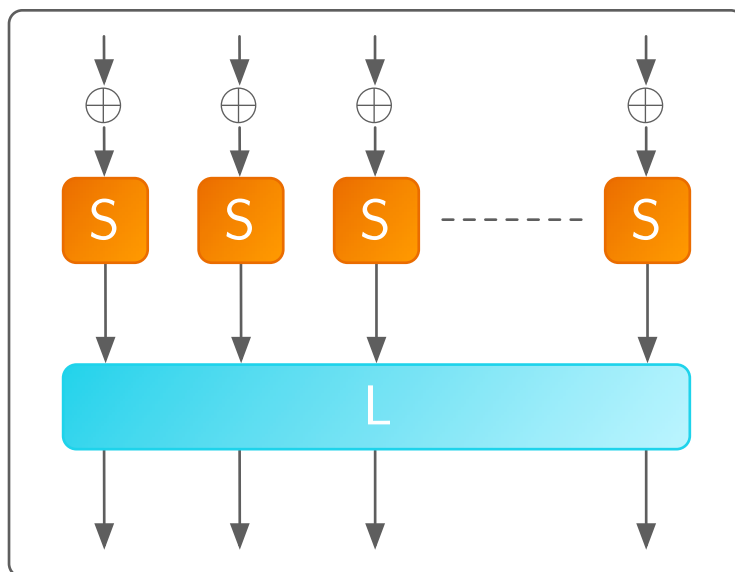


Dial C for Cipher

Description of **C**

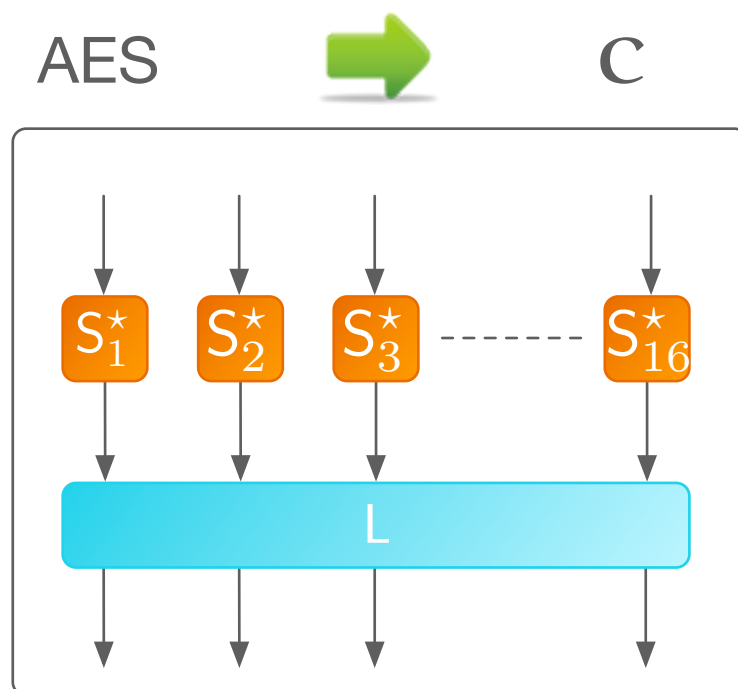
C corresponds to the AES where “addRoundKeys $\rightarrow$ SubBytes” is replaced by mutually independent random permutations.

AES



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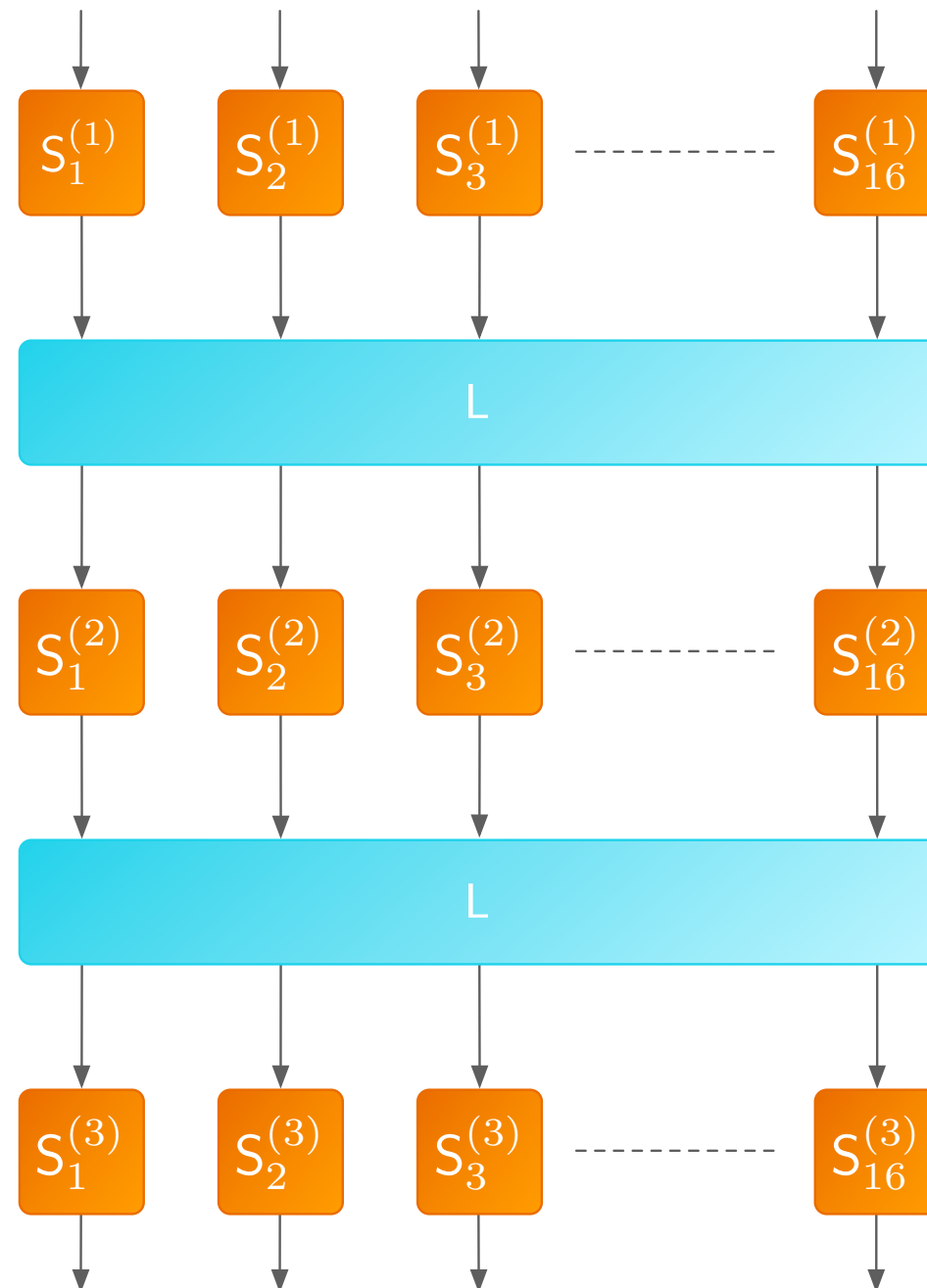
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- **C** is made of 9 identical rounds, followed by a layer of substitution boxes.
- **C** uses $16 \cdot 10 = 160$ mutually independent random 8-bits substitution boxes

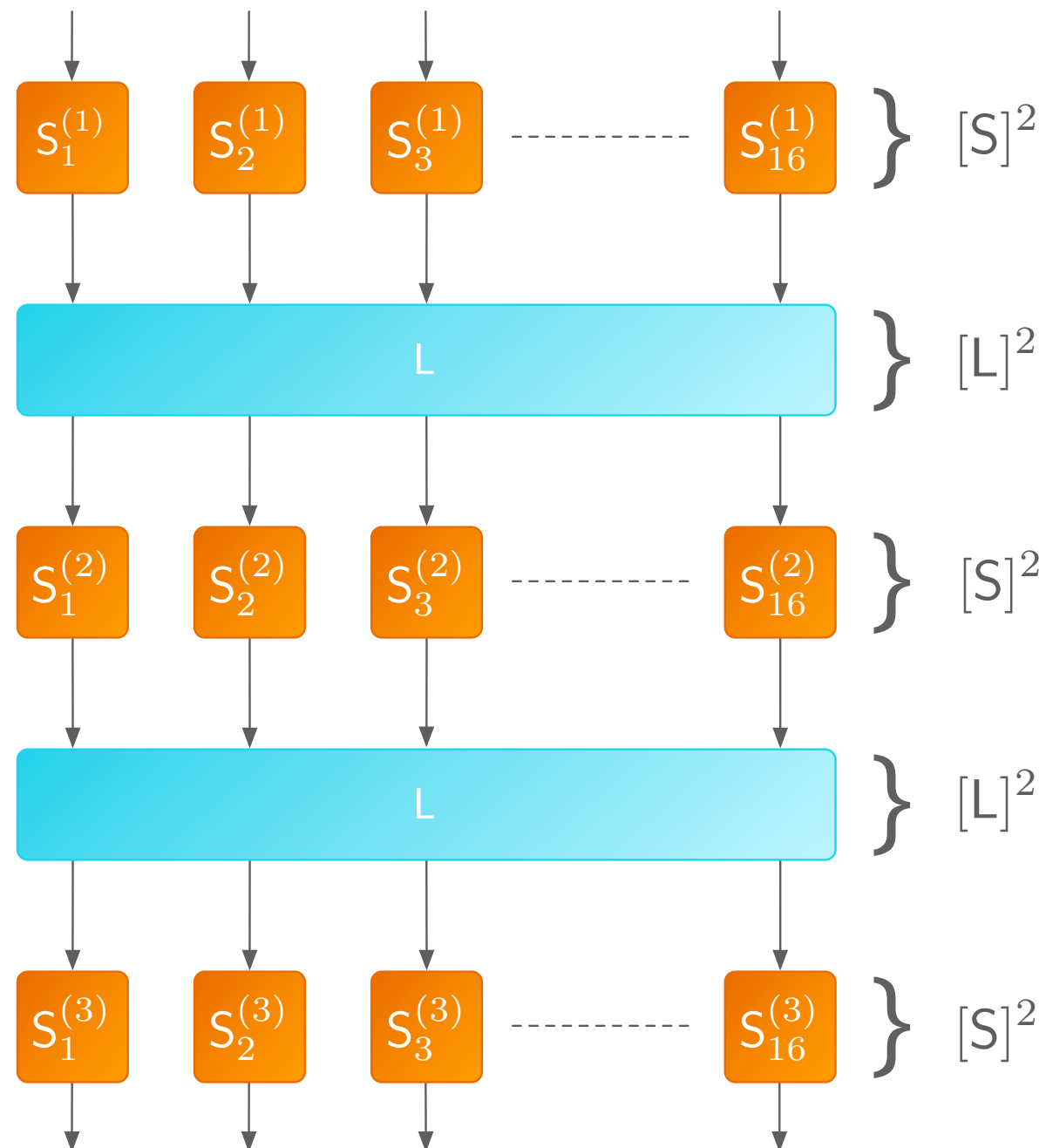
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We consider a version of $\mathbf{C}$ reduced to 3 rounds:



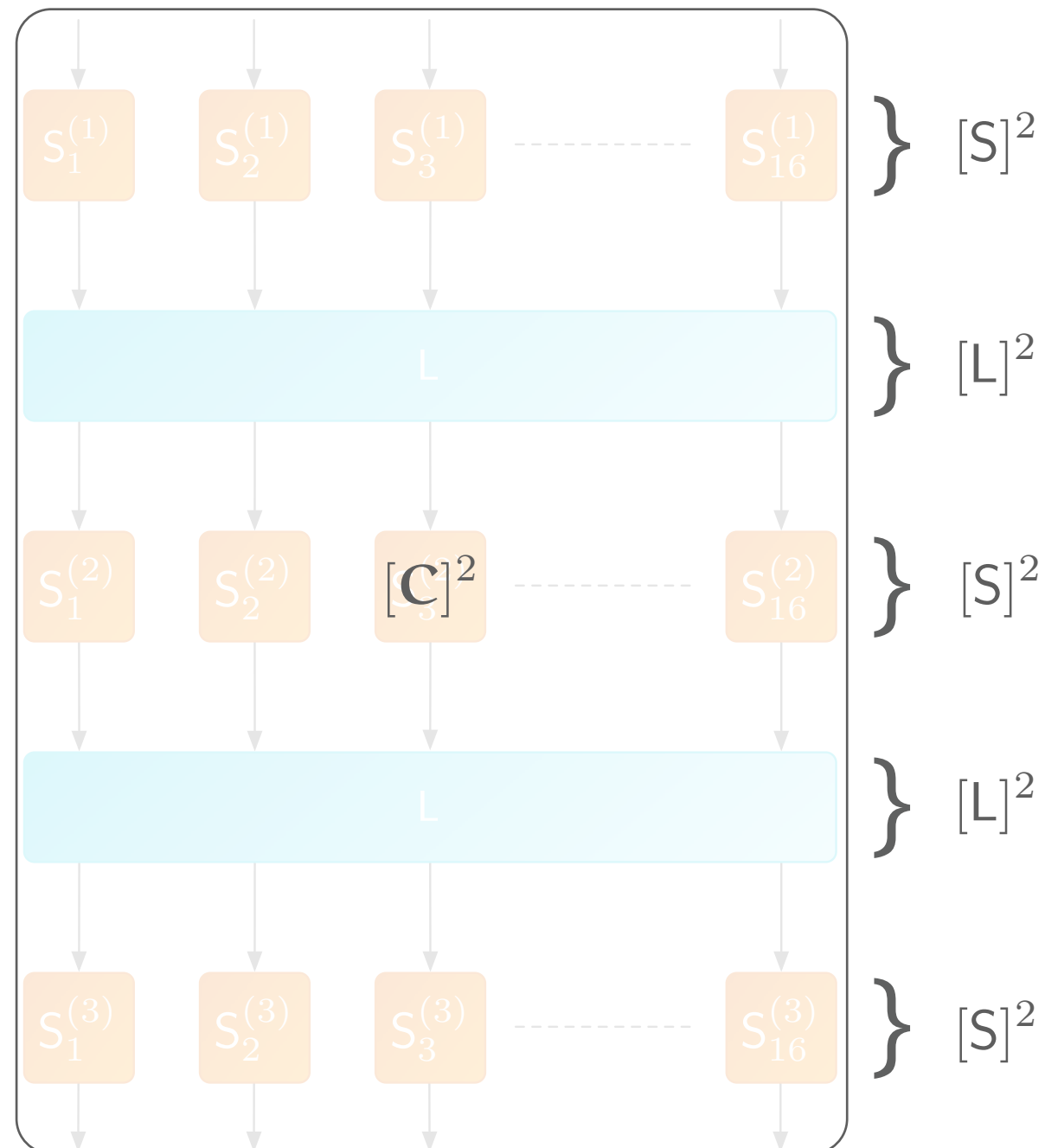
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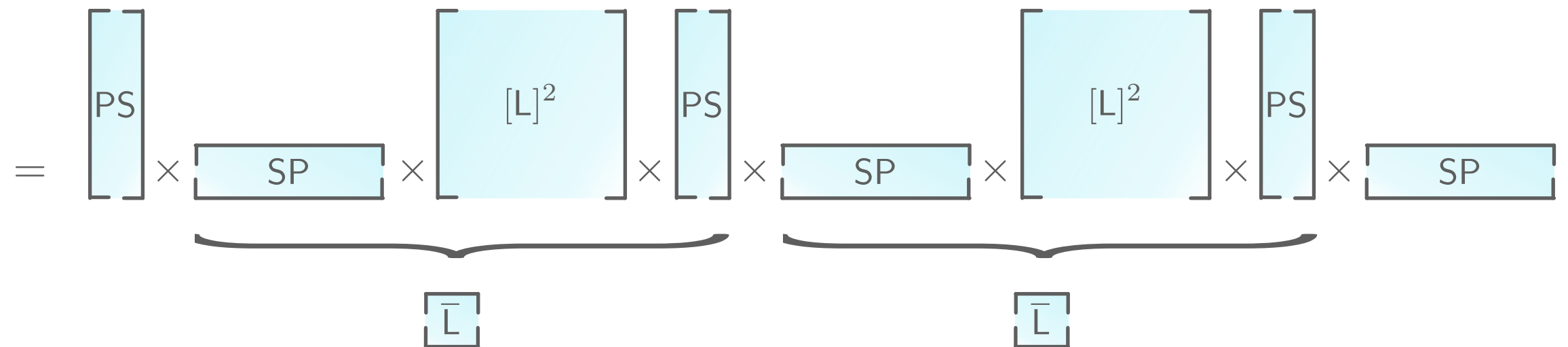
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where $\bar{\mathbf{L}}$ is a $2^{16} \times 2^{16}$ matrix.

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Computing the advantage of the best distinguisher (either adaptive or not) only requires operations on 625×625 matrices (instead of $2^{256} \times 2^{256}$ initially).

Values of $\text{Adv}_{\mathcal{A}}(\mathbf{C}, \mathbf{C}^*)$

r	1	2	3	4	5	6
$\text{Adv}(\mathbf{C}, \mathbf{C}^*)$	1	1	$2^{-4.0}$	$2^{-23.4}$	$2^{-45.8}$	$2^{-71.0}$
r	7	8	9	10	11	12
$\text{Adv}(\mathbf{C}, \mathbf{C}^*)$	$2^{-126.3}$	$2^{-141.3}$	$2^{-163.1}$	$2^{-185.5}$	$2^{-210.8}$	$2^{-238.9}$

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7 rounds of $\mathbf{C}$ are enough to obtain provable security against 2-limited adversaries

Part II: Designs and Security Proofs



KFC: the Krazy Feistel Cipher

What about Higher Orders?

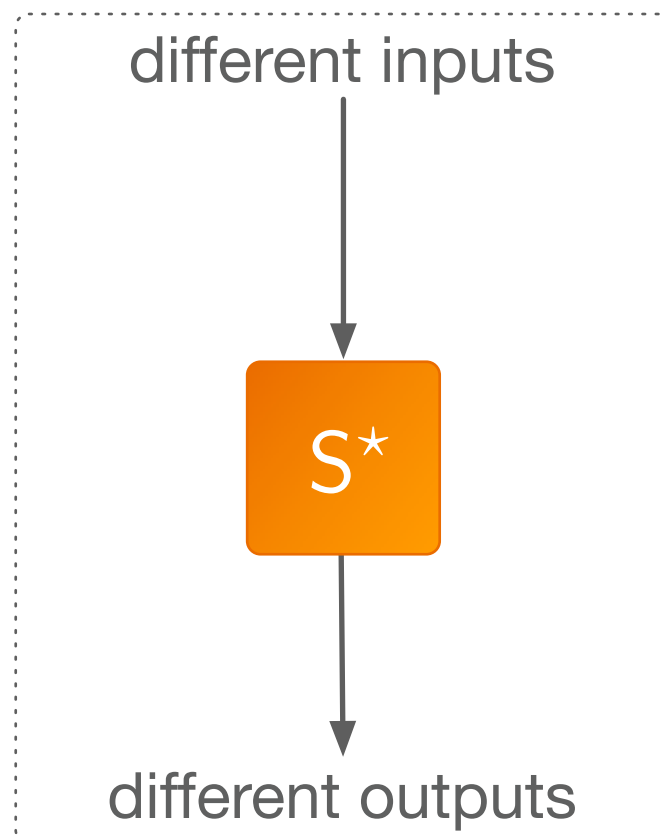
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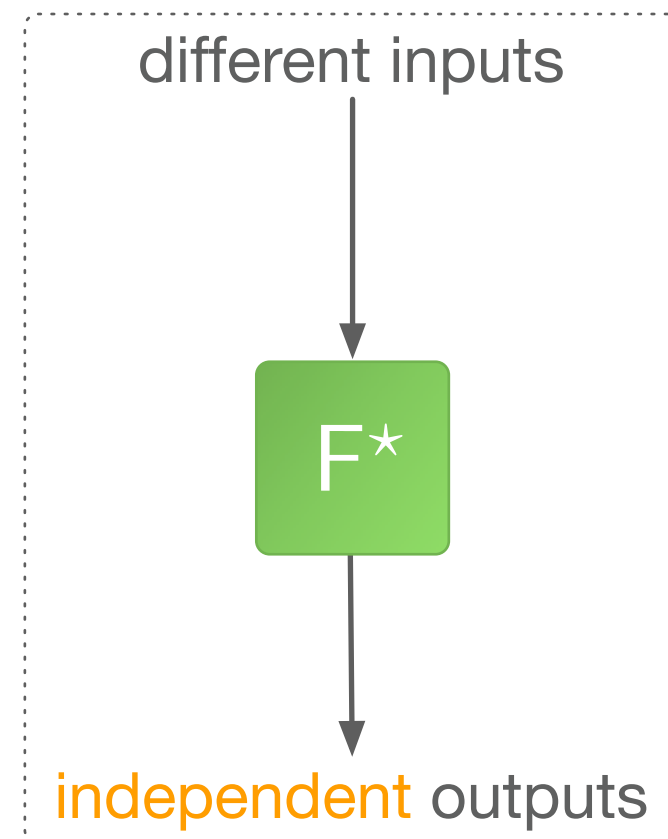
Idea: try to bound the advantage of the best q -limited adversary by that of the best $(q-1)$ -limited adversary.

Perfectly random permutation

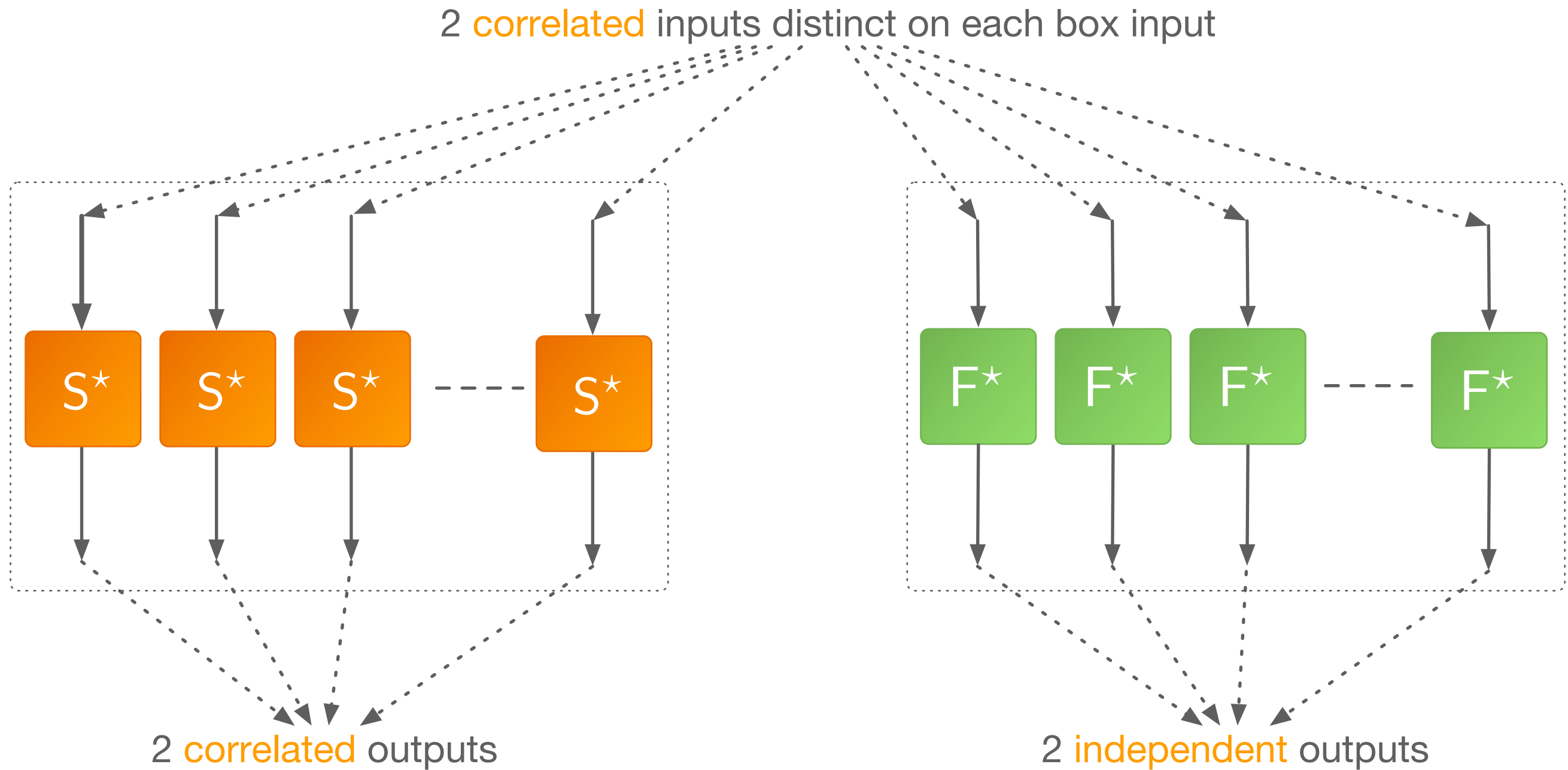


vs.

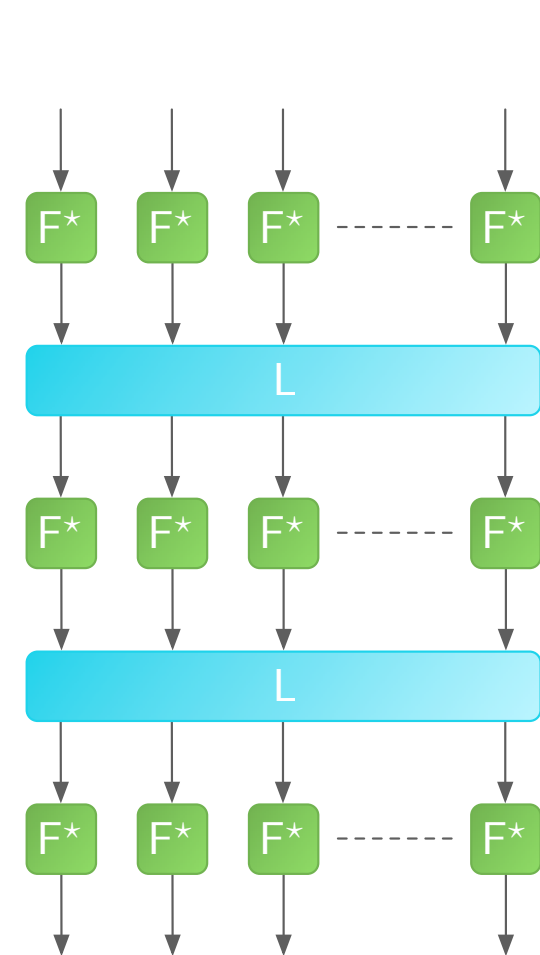
Perfectly random function



Rand. Permutations vs. Rand. Functions

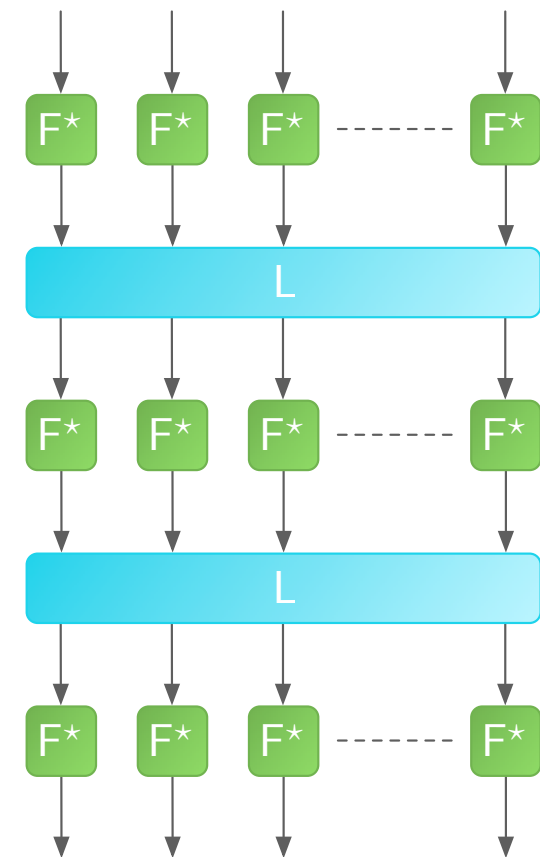


Towards a New Construction



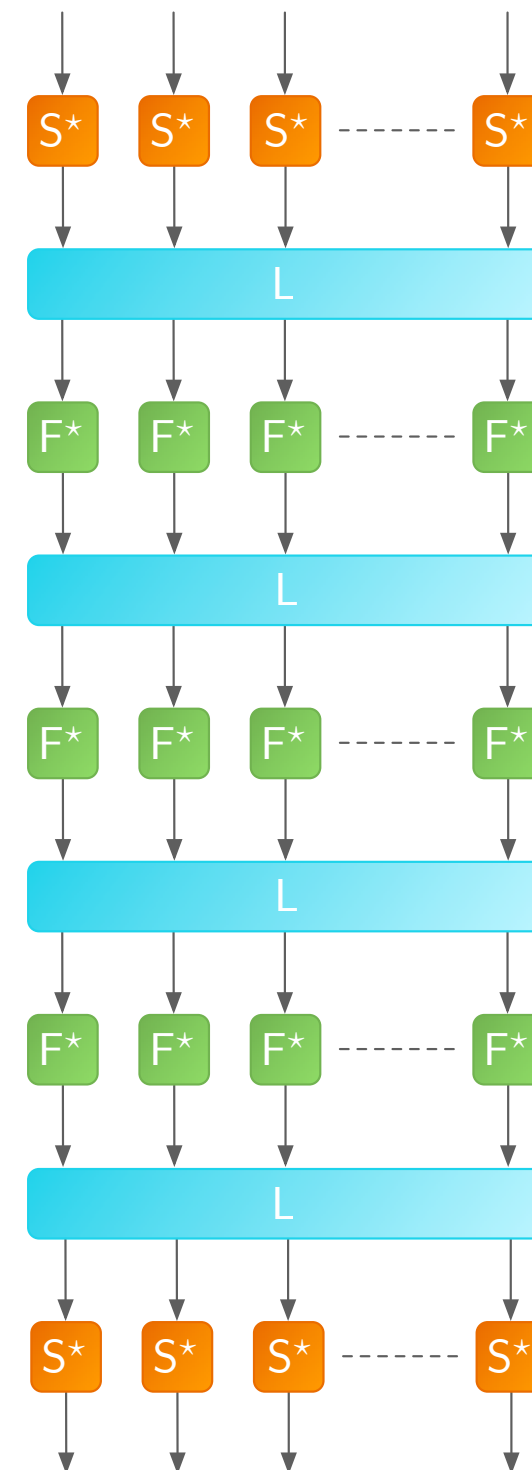
Towards a New Construction

- Non negligible risk of collision after a F-box



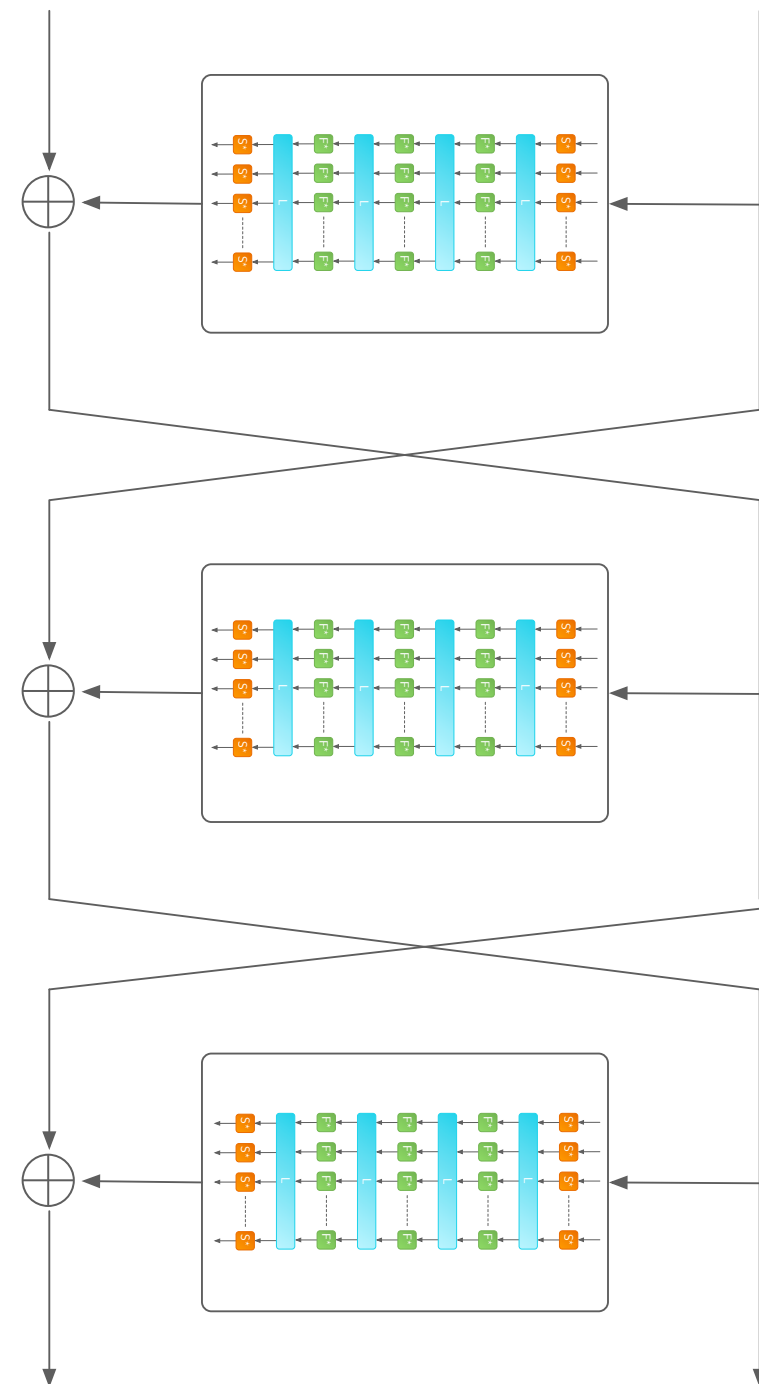
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- Use the “sandwich technique” to obtain (almost) pairwise independent inputs before the layer of random functions.
- The construction is not invertible. We plug it in a Feistel scheme.



Results obtained on KFC

- With this approach, we manage to prove the security against adversaries up to the order 70 (for an unreasonable set of parameters).
- The bounds are not tight at all it is certainly possible to improve our results.

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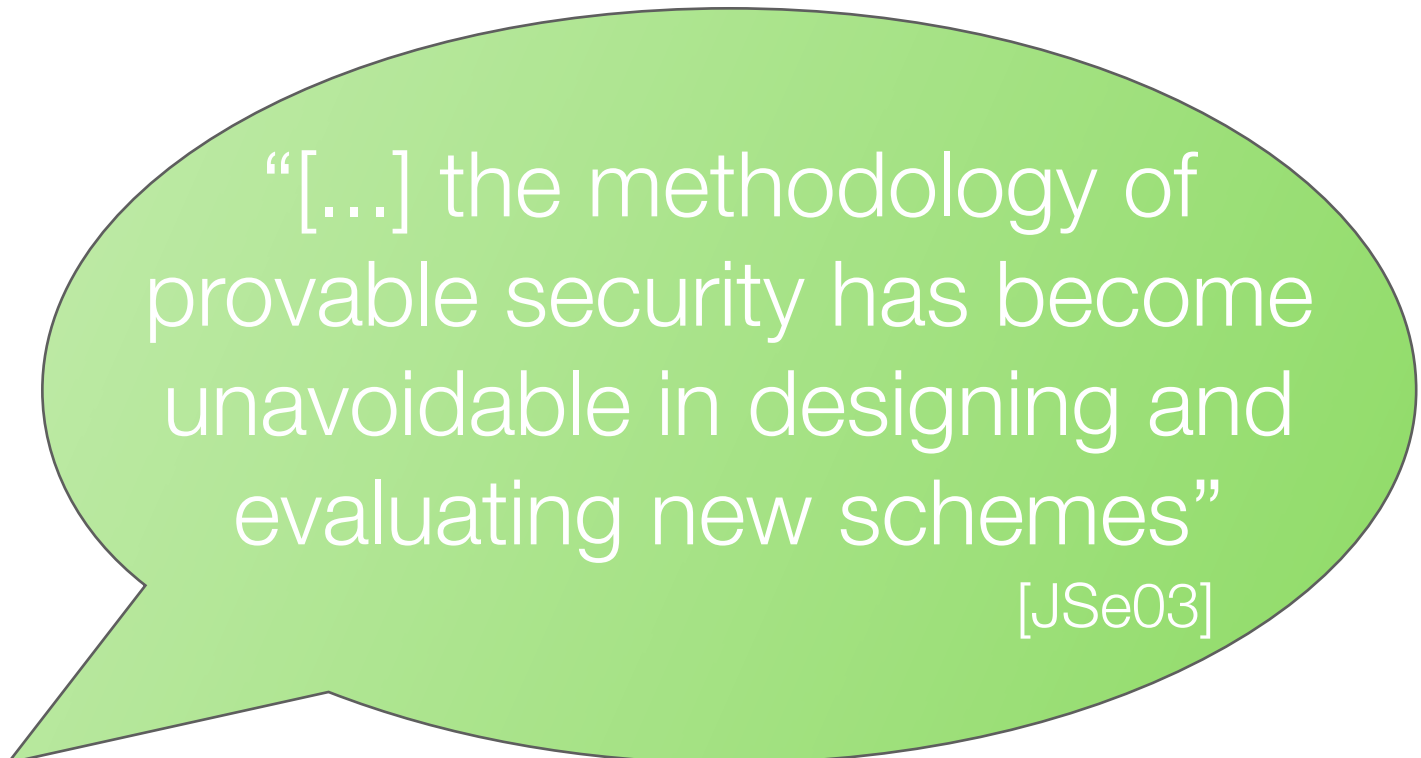
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Conclusion



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[JSe03]



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public key schemes

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public key schemes

We hope to have made a significant step towards its extension to block ciphers!

Thank you for your attention!



Publications

[BVicits08] *The Complexity of Distinguishing Distributions*

Joint work with [Serge Vaudenay](#)

Published in the proceedings of ICITS 08 (Calgary, Canada)

[BSVsac07] *Linear Cryptanalysis of Non Binary Ciphers (with an application to SAFER)*

Joint work with [Jacques Stern](#) & [Serge Vaudenay](#)

Published in the proceedings of SAC 07 (Ottawa, Canada)

[BFa06] *KFC - The Krazy Feistel Cipher*

Joint work with [Matthieu Finiasz](#)

Published in the proceedings of Asiacrypt 06 (Shanghai, China)

[BFsac06] *Dial C for Cipher*

Joint work with [Matthieu Finiasz](#)

Published in the proceedings of SAC 06 (Montreal, Canada)

[BVsac05] *Proving the Security of the AES Substitution-Permutation Network*

Joint work with [Serge Vaudenay](#)

Published in the proceedings of SAC 05 (Kingston, Canada)

[BJVa04] *How Far Can We Go Beyond Linear Cryptanalysis?*

Joint work with [Pascal Junod](#) & [Serge Vaudenay](#)

Published in the proceedings of Asiacrypt 04 (Jeju Island, Korea)

